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# REPAIRABLE ITEM INVENTORY SYSTEMS: A LITERATURE REVIEW

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**ABSTRACT**

This review covers the materials found in recent literature for the management of repairable item inventory systems. We provide a classification according to the modelling approach used (i.e., continuous review, periodic review, queueing-based models, others) and according to the structure of the inventory system involved (i.e., single or multi-echelon systems).

## 1. Introduction

This review is motivated by a study being done concerning the management of repairable items at a University. These items include electric fans, air-conditioners, typewriters, classroom chairs, and so on.

Earlier studies on the subject were motivated by the realization that recoverables form a substantial share in the total inventory investment of an organization. For instance, in the U.S. Navy, this figure represents approximately 58% of the total dollar investment in inventory [Schrady, 1967]. In the U.S. Air Force, the share of repairable items in the total spare parts inventory rose from 52% in 1968 [Sherbrooke, 1968] to approximately 65% in 1973 [Muckstadt, 1973]. Although the University counterpart to this figure is not available at present, awareness of the "repair-while-possible" policy that is generally in effect there more than adequately supports the importance of studying the system in the context of repairable items.

A thorough review of the literature on the subject, covering materials that date up to 1979 is in Nahmias (1981). For its part, the present survey includes the models found in the literature from 1979 to 1990. Reference to earlier work will still be made, however, where necessary.

A good part of the repairable item inventory literature is devoted to multi-echelon systems, i.e., those that are characterized by a significant interaction among the different production and storage facilities. On account of the fact that most repairable items have low demand and high value, the (S-1,S) type of ordering policy is often used; that is, when the inventory position drops below S, a replacement is immediately ordered to bring it back to S. However, situations do exist in which a one-for-one ordering policy is no longer appropriate, e.g., when ordering cost is greater than holding cost, in which case batch policies are more economical. Queueing theory also plays an important role in the management of recoverable items. This approach is invoked upon when repair capacity limitations exist. Finally, when the situation becomes too complex for modelling in terms of the preceding approaches, one resorts to heuristic and simulation techniques.

This review is organized according to the classification outlined in Table 1. We cover single-echelon models in Section 2, differentiating them by the type of policy used, namely, continuous review, periodic review, queueing-based models, and those that are characterized by rules that are not directly identifiable with the foregoing policies. In Section 3, we report on multi-echelon models, using the same four-way classification.

| | Single-echelon | Multi-echelon |
|---|---|---|
| Continuous review | | |
| (S-1,S) | Schaefer (1983) | Sherbrooke (1968), Muckstadt (1973), Shanker (1981), Muckstadt & Isaac (1981), Singh, Shah & Prem Vrat (1984), Kharola & Prem Vrat (1986), Graves(1985), Lee(1987) |
| Batch | Schrady (1967), Nahmias & Rivera (1979), Muckstadt & Isaac (1981) | Howard (1984), Moinzadeh & Lee (1986) Lee & Moinzadeh (1987) Schaefer (1989) |
| Periodic review | Lawrence & Schaefer (1984), Graves (1988) | |
| Queueing-based | Hillier & Lieberman(1980) Starr & Miller (1981), Scudder (1984), Taha (1987) Balana, Gross & Soland (1989) | Gross, Miller & Soland (1983), Gross & Miller (1984) |
| Others (heuristics, simulation, etc.) | Scudder & Hausman (1982) Matta (1985) Soni (1988) | Pyke (1990) |

Table 1. Summary of repairable item inventory literature included in this survey.

## 2. Single-echelon Systems

A single-echelon repairable inventory system typically consists of a repair shop and a store that keeps serviceable items and non-serviceable ones due for repair. The system is usually modelled in the context of: (1) a warehouse-repair facility system that fills requests for items coming from the rest of the organization, or (2) a maintenance center servicing a fixed number of machines that are subject to failure. In any case, two sources of serviceable items (machine parts, in the second case) are possible here: external purchases and returns from the repair shop.

### 2.1. Continuous Review Models

#### 2.1.1. One-for-one Replenishment Policy

Schaefer (1983) situates her model at a maintenance center for a finite number of machines that undergo periodic overhauls. The maintenance center follows an (S-1,S) policy for its parts inventories and allows no backorders for the same. The system is modelled using three alternative job-completion based criteria to determine inventories of multiple parts that are repairable. Exact and approximate solution approaches are proposed to solve the models.

#### 2.1.2. Batch Procurement and/or Repair Policy

One of the earliest studies involving batch policies in single-echelon repairable inventory systems is that of Schrady (1967). He develops a deterministic continuous review model that accounts for the interaction between repair and purchasing. The latter is actually a consequence of the positive scrapping rate and fixed number of items assumed for the system. This model is extended by Nahmias & Rivera (1979) to allow for a finite repair rate at the repair depot, instead of the infinite rate assumed by Schrady. Formulas expressing optimal batch repair and procurement quantities are derived in both papers, from cost functions that balance holding cost against ordering and repair-induction costs.

Muckstadt & Isaac (1981) formulate a cost model for a single item inventory system in which the demand for serviceable units and the returns of repairable units occur probabilistically. The cost function includes purchase ordering, holding, and backorder costs. An (s,Q) policy is assumed with respect to purchase decisions; that is, an order for Q units is placed as soon as the inventory position drops to s. Repairs are considered to take place continuously on a first-come-first-served basis, so that the quantities of interest are only s and Q. The optimal policy is then determined using a normal approximation to the stationary distribution of net inventory.

Schaefer (1989) proposes a method of determining optimal (s,S) purchase replenishment policies, treating the depletion and replenishment process as a Markov chain. We know that an (s,S) policy is equivalent to an (s,Q) policy if we let S=s+Q. Schaefer's model has the objective of minimizing the sum of the expected replenishment, holding, shortage and repair costs. The model captures repair characteristics by providing a general representation for the expected shortage and repair costs. A simple formulation that allows no batching for repair is given to illustrate the calculation of this cost.

**2.2. Periodic Review Models**

In Lawrence & Schaefer (1984), a model is developed to determine the optimal repairable parts inventory for a maintenance center servicing fault tolerant machines. Markov analysis is used to determine the steady state probabilities and optimal solutions are obtained by Dynamic Programming.

Graves (1988) treats the spares allocation problem in conjunction with repair capacity requirements to meet a particular service strategy for the field support of an office equipment product. He proposes a discretetime linear systems model characterizing the tradeoff between the inventory investment in spare components and the cost of the work force.

### 2.3. Queueing-based Models

The foregoing models assume ample repair capacity such that no queues are expected to form in the system. When repair capacity is limited, the problem translates to a queueing type one where, in addition to spares levels, the question of repair capacities is also of concern.

Starr & Miller (1981) point out the similarities between queueing and inventory systems, and show how queueing theory can be applied to inventory control.

A single-echelon repairable inventory system can also be viewed in terms of the classical machine repair problem. In its simplest form, the system has one level of resupply and one repair facility, and can be considered as a finite source queue (of M machines, say), or a closed queueing network. The problem is treated in practically every book in Operations Research [see, e.g., Hillier & Lieberman (1980), and Taha (1987)].

Balana, Gross & Soland (1989) treat the problem in a nonequilibrium environment, in which repair and failure rates are allowed to vary over time. They present an algorithm that uses an implicit enumeration method to find the least cost mix of spares and repair channels.

### 2.4. Others

Soni (1988) extends the machine repair model to incorporate such characteristics as scrapping and variable number of operating channels. Two models are presented and efficient solution procedures (based on iterative aggregate/disaggregate techniques) are developed to obtain the stationary distribution of the Markov models. The steady state distribution may then be used within an optimization model to determine the optimal values of the variables of interest: e.g., spares inventories and the required number of repairmen.

Practical considerations of limited repair capacity and prioritized shop dispatching rules combine to make repair times dependent on one another. Scudder & Hausman (1982) study this case using a simulation model to explore a variety of heuristic approaches to the spares stocking decision for a repair facility, under conditions of limited capacity and prioritized scheduling.

Matta (1985) presents a GASP IV based simulation model that captures the dynamic and stochastic character of repairable items/spare parts inventory systems. The simulation model has the capability of investigating the influence of an order quantity or an order level inventory policy, the number of repair stations and the time between reviews, on shortages, utilization and cost.

## 3. Multi-echelon Systems

Multi-echelon systems can be typically represented by a set of sites supported by a centrally located depot. The problem is that of determining the appropriate inventory levels of spares at each location. Item failures occur at the base level and if possible, repair is carried out there; otherwise, the failed items are sent to the depot (for repair), from which a replacement is promptly requested. The multi-echelon structure, therefore, may apply not only to the stores but also to the repair facilities.

### 3.1. Continuous Review Models

#### 3.1.1. One-for-one Replenishment Policy

Two of the important developments in this problem area are reported upon by Sherbrooke (1968) and by Muckstadt (1973). Sherbrooke's model, called

METRIC (Multi-Echelon Technique for Recoverable Item Control), represents a 2-echelon structure of storage and of repair with unlimited repair capacity. The model seeks to find the inventory positions at all locations with the objective of minimizing the total number of expected backordered units at the bases subject to a spares budget constraint. Muckstadt extends the METRIC model to account for a hierarchical parts structure. His so-called MODMETRIC formulation considers two levels of indenture representing the end item and the modules that comprise it. The model minimizes the total expected backorders for the end item, subject to a budget constraint on both the end item and module spares.

In Singh, Shah & Prem Vrat (1980), we find a mathematical model for allocating expensive spares to different locations in a repair inventory system consisting of a divisional store and workshop on the first level and a number of depots on the second. Repairs take place only on the first echelon and a one-for-one stock replacement policy holds on the second. The model, designed for a real life Transport Corporation, relates the echelon inventory with the total system cost, inclusive of holding, shortage, and transportation costs. An iterative solution methodology is proposed.

Graves (1985) considers a system consisting of a central repair depot that supports a number of operating sites. He presents an exact model for obtaining the steady state distribution of the net inventory at each site.

Muckstadt & Isaac (1981) study the case in which the demand rate exceeds the return rate from the repair facility, a situation that calls for procurement from an outside source. They model the problem in the context of a warehouse with repair and storage facilities and a number of lower echelon retailers with storage facilities. They develop a cost model based on an (S-1,S) policy on the second level and an (s,Q) policy on the first. The solution method is an extension of that which was developed for the single-echelon case, also described in the same paper.

Shanker (1989) analyzes a 2-echelon system with two levels of supply and two levels of repair. Allowing for the condemnation of non-recoverable items and assuming an (S-1,S) procurement policy at the bases and an (s,S) policy at the depot, he derives exact expressions for stationary distributions of the inventory position at the depot, and of the number of backorders, the on-hand inventory and the in-repair inventory at each location, based on deterministic repair and lead times.

Lee (1987) considers a ramification to the multi-echelon case where emergency lateral transshipments are allowed. He uses the expected number of backorders and the quantity of lateral transshipments as the key performance measures, and derives approximate expressions for these, which are then used to determine optimal stocking levels.

Kharola & Prem Vrat (1986) integrate the recoverable item inventory problem with that of repair capacity requirement and preventive maintenance timing. Their inventory sub-model imposes a one-for-one replenishment policy on the lower level and allows no lateral resupply of items. An iterative solution methodology is decribed for the model.

### 3.1.2. Batch Replenishment Policy

Batch ordering is usually called for when the system experiences high demand rates and/or incur ordering costs that are relatively high. Howard (1984) describes a model in the context of provisioning collieries with spares and repairable sub-assemblies for its machinery. He shows how standard stock control theory can be applied to the case where stock replenishments can be effected by purchasing and repair. Whereas the system of concern basically consists of two stockage levels and one repair level, his formulation focuses on batch decisions on level 1, that is, the central store that interacts with the repair shop and the colliery stores.

Moinzadeh and Lee (1986) present a model that minimizes the sum of set-up, inventory holding and backorder costs. A power approximation method is employed to estimate the total system stock and backorder levels. These estimates are then used to find the optimal batch size which, in turn, determines the optimal stocking levels at the depots and sites.

Lee & Moinzadeh (1987) propose a two-parameter approximation scheme to determine the distribution of the number of outstanding orders and the backorders developed. Their model is, in fact, a generalization of the (S-1,S) policy and their approach is an extension of that of Graves (1985) for the batching case.

### 3.2. Queueing-based Models

Gross, Miller & Soland (1983) study the tradeoffs between spares levels and repair capacities in a repairable-item provisioning system with two levels of repair and one level of supply. The system is modelled as a closed queueing network and an implicit enumeration algorithm is used to solve the optimization problem.

Gross & Miller (1984) consider the transient behavior of the system and develop Markov models both for one and two levels of repair and supply. The randomization technique of obtaining transient solutions to the Markov models is also described.

### 3.3. Others

Pyke (1990) treats the stock allocation problem as part of a complex model for a repairable item logistics system composed of a depot and several bases that respond to the parts needs of military aircraft. He investigates the use of priority rules for repair at the depot and for the distribution of repaired items to the bases. The objective is to minimize the number of grounded aircraft. The model, complicated by the presence of priority rules, lateral resupply, and canibalization, is solved by simulation.

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