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A discrete–time batch Markovian arrival process as B-ISDN traffic model

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Abstract

In this paper, a class of versatile discrete–time Markovian arrival processes (D-BMAP's) is introduced. A model for the superposition of video sources, both with uniform and multiple activity levels, belongs to this class. Formula for the correlation between arrivals in the D-BMAP are derived.
Observing the D-BMAP/G/1/N queue at departure epochs results in a finite Markov chain of M/G/1 type. An efficient method allowing the computation of the queue lenght distribution at departures is proposed. Using matrix analytical methods, we derive a solution for the buffer occupancy and for the loss probability which can be expressed in a form suitable for numerical computations. Finally, we show that the output process of this queue is again a DMAP.

Keywords : Markovian arrival process, traffic model, finite Markov chain, finite capacity queue

1 Introduction

Integrated broadband communication systems must offer a high degree of flexibility, together with efficiency in resource consumption, by sharing the same network resources (bandwidth, buffers,...) among several connections (data, voice, video,...). These connections generate traffic streams with very different characteristics (required bandwidth, burstiness, correlation, ...). Each connection has its own specific quality of service requirements with respect to delay, delay jitter and loss of information. The efficiency in bandwidth usage is increased by taking advantage of the statistical fluctuations in bandwidth requirements of the individual connections. For the design and the dimensioning of the network components (buffer requirements in multiplexers and switches, connection acceptance control strategies, source policing mechanisms,...) an analytical model describing the traffic is essential. Such an analytical source model has to fulfill a number of criteria, such as generality, accuracy, close to reality and computational tractability.

Commonly used models like the Poisson and Bernoulli process, in spite of their computational tractability, often do not incorporate important characteristics of the real traffic (e.g. burstiness, periodicity, etc.; see [19]). Consequently there is a need for more detailed models which are still analytically tractable.

In earlier contributions [2], [3], a class of general discrete-time Markovian arrival processes (DMAP's) was introduced. In [12] it is shown that, as model for the superposition of sporadic sources, the DMAP gives rise to a rather poor accuracy when evaluating the performance of the corresponding multiplexer (the loss curve does not show the sharp bend at the transition from cell level to burst level statistics).

Therefore, we propose a new model, called the *Discrete-time Batch Markovian Arrival Process* (D-BMAP). It is a discrete-time version of the versatile Markovian point process, introduced by Neuts [17] and more recently called BMAP by Lucantoni [15]. We show that this rich class of arrival processes includes many well-known source models as special cases. In particular, it is shown that an approximate model for the superposition of sporadic sources belongs to the class of D-BMAP processes. Furthermore we show that the process introduced in [16] to model a superposition of video sources with uniform activity levels and the process in [20] modeling video sources with multiple activity levels, both belong to the class of D-BMAP processes.

As the correlation structure of the traffic stream in an ISDN network has a crucial influence on the quality of service of the connections (see e.g. [19]), we derive a formula for the correlation between arrivals in the D-BMAP.

The related statistical multiplexer can be modeled as a D-BMAP/G/1/N queue. Observing this queue at departure epochs results in a finite Markov chain of M/G/1 type. An efficient method allowing the computation of the queue lenght distribution at departures is proposed. Using matrix analytical methods, we derive a solution for the buffer occupancy and for the loss probability which can be expressed in a form suitable for numerical computations. Finally, we show that the output process of this queue is again a DMAP. The paper is organized as follows. In Section 2, the class of D-BMAP's is defined, examples of processes belonging to this class are given and the correlation structure is investigated. In Section 3, examples are given of traffic sources which can be modeled by a D-BMAP. Section 4 deals with the discrete-time D-BMAP/G/1/N queue. In Section 5, conclusions are drawn and topics for further research are given.

2 A Discrete-Time Batch Markovian Arrival Process

In this section we define the Discrete-Time Batch Markovian Arrival Process (D-BMAP). It is the discrete-time analogue of the versatile Markovian point process introduced by Neuts [17] and recently studied using a more transparent notation by Lucantoni [15]. It is shown that this rich class of discrete-time arrival processes contains many processes useful as source model for analytical studies in network dimensioning and design problems. We also give formulas for the correlation between arrivals of the D-BMAP.

2.1 Definition

In order to better understand the evolution of the process, we start by giving a constructive description of the process. Consider a discrete-time Markov chain with transition matrix $\mathbf{D}$. Suppose that at time k this chain is in some state i, $1 \leq i \leq m$. At the next time instant $k+1$, there occurs a transition to another or possible the same state and a batch arrival may or may not occur. With probability $(d_0)_{i,j}$, $1 \leq i \leq m$, there is a transition to state j without an arrival, and with probability $(d_n)_{i,j}$, $1 \leq i \leq m$, $n \geq 1$, there is a transition to state j with a batch arrival of size n. We have that

$$\sum_{n=0}^{\infty} \sum_{j=1}^{m} (d_n)_{i,j} = 1.$$

Clearly the matrix $\mathbf{D}_0$ with elements $(d_0)_{i,j}$ governs transitions that correspond to no arrivals, while the matrices $\mathbf{D}_n$ with elements $(d_n)_{i,j}$, $n \geq 1$, govern transitions that correspond to arrivals of batches of size n.

More formally, the process can be defined as a two dimensional discrete-time Markov process $\{(N(k), J(k)), k \geq 0\}$ on the state space $\{(n,j), n \geq 0, 1 \leq j \leq m\}$ with transition matrix

$$\mathbf{T} = \begin{pmatrix} \mathbf{D_0} & \mathbf{D_1} & \mathbf{D_2} & \mathbf{D_3} & \dots \\ \mathbf{0} & \mathbf{D_0} & \mathbf{D_1} & \mathbf{D_2} & \dots \\ \mathbf{0} & \mathbf{0} & \mathbf{D_0} & \mathbf{D_1} & \dots \\ \vdots & \vdots & \ddots & \ddots & \dots \\ \vdots & \vdots & \ddots & \ddots & \dots \end{pmatrix}.$$

The variable $\{N(k), k \geq 0\}$ represents the counting variable and $\{J(k), k \geq 0\}$ the phase variable. With this notation, the transition from state (l, i) to state $(l+n, j)$ corresponds to an arrival of size n and a phase change of i to j.
The matrix $\mathbf{D} = \sum_{n=0}^{\infty} \mathbf{D_n}$ is the transition matrix of the underlying Markov chain . Let $\overline{\pi}$ be stationary probability vector of this Markov process, i.e.

$$\overline{\pi}\,\mathbf{D} = \overline{\pi}, \quad \overline{\pi}\,\overline{\mathbf{e}} = 1,$$

where $\overline{\mathbf{e}}$ is a column vector of 1's.
The fundamental arrival rate λ of this process is given by

$$\lambda = \overline{\pi}\,(\sum_{k=1}^{\infty} k\mathbf{D}_k)\,\overline{\mathbf{e}}.$$

2.2 Correlations between Arrivals

As shown by Ramaswami and Willinger [19], the correlation structure of traffic streams is an essential characteristic which influences heavily the quality of service.
Let $(X_1, \dots, X_k)$ be a set of random variables, where X_i is the number of arrivals at time instant i. $\mathbf{f}(x_1, \dots, x_k)$ denotes the joint distribution matrix of $(X_1, \dots, X_k)$, i.e. $f_{ij}(x_1, \dots, x_k)$ is the conditional probability that $X_1 = x_1, \dots, X_k = x_k$ and that the phase of the arrival process at time instant k is j, given that the process started in phase i at time 0.
Denote $\tilde{\mathbf{f}}(z_1, \dots, z_k)$ the corresponding z-transform. Then clearly

$$\tilde{\mathbf{f}}(z_1, \dots, z_k) = \mathbf{D}(z_1) \dots \mathbf{D}(z_k), \tag{1}$$

where $\mathbf{D}(z) = \sum_{i=0}^{\infty} \mathbf{D}_n z^n$.
The correlation between two random variables X_1 and X_k is expressed in terms of their

covariance matrix

$$\mathbf{COV}(X_1X_k) = \sum_{x_1=0}^{\infty}\cdots\sum_{x_k=0}^{\infty}(x_1-\mu_1)(x_k-\mu_k)\mathbf{f}(x_1,\ldots,x_k) \tag{2}$$

$$= \mathbf{E}[X_1X_k] - \mu_1\mathbf{E}[X_k] - \mu_k\mathbf{E}[X_1] + \mu_1\mu_k\mathbf{f}(1,\ldots,1), \tag{3}$$

with μ_1 and μ_k the scalar mean of X_1 and X_k. The scalar covariance function is given by

$$\begin{aligned} COV(X_1X_k) &= \overline{\boldsymbol{\pi}}\mathbf{COV}(X_1X_k)\overline{\mathbf{e}} \\ &= \overline{\boldsymbol{\pi}}\mathbf{E}[X_1X_k]\overline{\mathbf{e}} - \mu_1\mu_k \end{aligned} \tag{4}$$

Recall that the coefficient of correlation between two variables X_1 and X_k is defined as

$$c_{corr}(X_1X_k) = \frac{COV(X_1X_k)}{\sigma(X_1)\sigma(X_k)}$$

where $\sigma^2(X_i)$ denotes the variance of X_i, $i = 1, k$.
The mean matrices $\mathbf{E}[X_1]$, $\mathbf{E}[X_k]$ and $\mathbf{E}[X_1X_k]$ are given by

$$\mathbf{E}[X_1] = \frac{\partial}{\partial z_1}\tilde{\mathbf{f}}(z_1,\ldots,z_k)\mid_{z_i=1} = [\sum_{l=1}^{\infty} l\mathbf{D}_l]\mathbf{D}^{k-1}, \tag{5}$$

$$\mathbf{E}[X_k] = \frac{\partial}{\partial z_k}\tilde{\mathbf{f}}(z_1,\ldots,z_k)\mid_{z_i=1} = \mathbf{D}^{k-1}[\sum_{l=1}^{\infty} l\mathbf{D}_l], \tag{6}$$

$$\mathbf{E}[X_1X_k] = \frac{\partial^2}{\partial z_1\partial z_k}\tilde{\mathbf{f}}(z_1,\ldots,z_k)\mid_{z_i=1} = [\sum_{l=1}^{\infty} l\mathbf{D}_l]\mathbf{D}^{k-2}[\sum_{l=1}^{\infty} l\mathbf{D}_l]. \tag{7}$$

For a stationary D-BMAP, the mean number of arrivals per time unit is given by the arrival rate λ, so that for μ_1 and μ_k we obtain

$$\mu_1 = \mu_k = \overline{\boldsymbol{\pi}}[\sum_{l=1}^{\infty} l\mathbf{D}_l]\mathbf{D}^{k-1}\overline{\mathbf{e}} = \lambda.$$

Consequently, the covariance matrix of X_1 and X_k is given by

$$\mathbf{COV}(X_1X_k) = [\sum_{l=1}^{\infty} l\mathbf{D}_l - \lambda\mathbf{D}]\mathbf{D}^{k-2}[\sum_{l=1}^{\infty} l\mathbf{D}_l - \lambda\mathbf{D}]. \tag{8}$$

For the scalar covariance function we obtain

$$COV(X_1, X_k) = \overline{\boldsymbol{\pi}}[\sum_{l=1}^{\infty} l\mathbf{D}_l]\mathbf{D}^{k-2}[\sum_{l=1}^{\infty} l\mathbf{D}_l]\overline{\mathbf{e}} - \lambda^2.$$

From this we derive the coefficient of correlation

$$c_{corr} = \frac{\overline{\boldsymbol{\pi}}[\sum_{l=1}^{\infty} l\mathbf{D}_l]\mathbf{D}^{k-2}[\sum_{l=1}^{\infty} l\mathbf{D}_l]\overline{\mathbf{e}} - \lambda^2}{\overline{\boldsymbol{\pi}}[\sum_{l=1}^{\infty} l^2\mathbf{D}_l]\overline{\mathbf{e}} - \lambda^2}.$$

3 The D-BMAP as Traffic Model

In this section we show by means of examples how the D-BMAP can be used as model for various traffic sources, in particular for the superposition of variable bit rate sources, such as still picture, compressed video, etc. For a more detailed discussion we refer to [5].

3.1 Special Cases of the D-BMAP

A number of well known discrete-time arrival processes can be obtained as special case of the D-BMAP.

(1) *The Discrete-time Markovian Arrival Process (DMAP)*
This process, defined in [2] and [3], is a D-BMAP with all arrivals having a batch of size 1. Some examples of D-BMAP's which are useful as traffic model are
• The Bernoulli Arrival Process
• The Discrete-Time Markov Modulated Bernoulli Process
• The Discrete-Time Markov Modulated Bernoulli Process with Minimum Interarrival Time.
(for more details we refer to [2], [3] and [6]).

(2) *A superposition of D-MAP's*
Consider two D-MAP's characterized by the matrices $\mathbf{D}_0^{(i)}$ and $\mathbf{D}_1^{(i)}$, i=1,2. Then the superposition is a B-DMAP, with matrices $\mathbf{D}_0 = \mathbf{D}_0^{(1)} \otimes \mathbf{D}_0^{(2)}$, $\mathbf{D}_1 = \mathbf{D}_0^{(1)} \otimes \mathbf{D}_1^{(2)} + \mathbf{D}_1^{(1)} \otimes \mathbf{D}_0^{(2)}$ and $\mathbf{D}_2 = \mathbf{D}_1^{(1)} \otimes \mathbf{D}_1^{(2)}$.

(3) *A D-MAP with i.i.d. batch arrivals*
Consider a D-MAP characterized by the matrices $\mathbf{D}_0$ and $\mathbf{D}_1$. Suppose that each arrival epoch corresponds to a batch arrival, where the successive batch sizes are independent and identically distributed with density $\{b_l,\ l \geq 1\}$. Then this process is a D-BMAP with matrices $\mathbf{D}_0$ and $\mathbf{D}_n = b_n\, \mathbf{D}_1$, for $n \geq 1$.

(4) *A batch Bernoulli process with correlated batch arrivals*
Consider a Bernoulli arrival process with parameter p, and suppose that each arrival epoch corresponds to a batch arrival. The batch arrival size distribution $\{q_i(.), 1 \leq i \leq m\}$ is governed by an m-state discrete-time Markov chain, with transition probability matrix $\mathbf{P}$. The process defined in this way is a D-BMAP, where $\mathbf{D}_0 = (1-p)\mathbf{I}$ and $(\mathbf{D}_n)_{ij} = p\mathbf{P}_{ij}q_i(n)$.

3.2 An approximation for the superposition of sporadic sources

In [2] it has been shown that the DMAP is able to model many sources, in particular sporadic sources. Since the superposition of DMAP's is a D-BMAP, we could in principle describe a superposition of sporadic sources by means of the appropriate D-BMAP (see Example (2) in 2.3). This approach involves a very large state space, and therefore we propose the following approximate model, belonging to the class of D-BMAP's.

Consider a sporadic source which generates packets at regular instants during an active period. The time between two consecutive packets during such an active period is supposed to be d time units. The duration of an active period is supposed to be geometrically distributed with mean p time units. An active period is followed by a silent period the duration of which follows a geometrical distribution, with mean q. In this way, a source is characterized by means of the triple (p, q, d).

Since the mean number of time units before an active source becomes silent is given by p, we immediately derive that the probability that an active source becomes silent in the next time unit is given by $\beta = 1/p$. Similarly, the probability that a silent source becomes active in the next time unit is given by $\alpha = 1/q$.

Now consider the stochastic process consisting of a superposition of a M identical independent sporadic sources with parameters (p, q, d) as defined above. The resulting arrival process has two important characteristics : firstly, it has a periodical character due to the deterministic character of the arrival process of a single active source and secondly, this process is modulated by the number of active sources. A detailed model, incorporating both characteristics, leads to a very large state space (for a more detailed discussion we refer to [5] and [13]). In order to avoid such a large state space, we shall simplify the model in the following way. We model the process of the number of active sources by means of an $(M+1)$-state pure birth and death process : we suppose that during a time unit only one source can change its state (active/silent). The transition matrix of this discrete Markov chain is then given by

$$\mathbf{D} = \begin{pmatrix} 1-M\alpha & M\alpha & 0 & \dots & 0 & 0 \\ \alpha & 1-\beta-(M-1)\alpha & (M-1)\alpha & \dots & 0 & 0 \\ 0 & 2\beta & 1-2\beta-(M-2)\alpha & \dots & 0 & 0 \\ 0 & 0 & & & & \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ 0 & 0 & 0 & \dots & M\beta & 1-M\beta \end{pmatrix}. \quad (9)$$

While this chain is in state m, the m active sources generate cells with a complex interarrival distribution, due to the fact that the cell interarrival time of an active source is deterministic, as has been observed above. In order to keep the number of states of the system limited, we suppose that the number of arrivals during a time slot only depends on the number of active sources. This means that the number of arrivals is renewed every time slot. Let $c_k(m)$ be the probability of k arrivals during a time slot when the Markov chain is in state m (i.e. $c_k(m)$ is the probability of having k arrivals during a time slot when m sources are active). Supposing that an active source has probability $1/d$ in each slot to generate a cell, then

$$c_k(m) = \begin{pmatrix} m \\ k \end{pmatrix} (\frac{1}{d})^k (\frac{d-1}{d})^{m-k}.$$

Let us now show that the resulting process is a D-BMAP.
Consider the $M \times M$ matrices $\mathbf{D}_n$ given by

$$\mathbf{D}_n = \mathbf{C}_n \, \mathbf{D}, \tag{10}$$

where $\mathbf{C}_n$ is the following diagonal matrix

$$\mathbf{C}_n = \begin{pmatrix} c_n(0) & & & & & \\ & c_n(1) & & & & \\ & & \cdots & & & \\ & & & c_n(i) & & \\ & & & & \cdots & \\ & & & & & c_n(M) \end{pmatrix}.$$

Then clearly the process defined above is a D-BMAP, characterized by the matrices $\mathbf{D}_n$.

3.3 Superposition of Variable Bit Rate Sources with Uniform Activity Levels

In [16], Maglaris et al. study the performance of a statistical multiplexer whose input consists of a superposition of full motion video sources with relatively uniform activity levels. The arrival process is modeled as a discrete-state continuous-time Markov process in the following way. The bit rate resulting from the superposition is quantized into finite discrete levels and the transitions between levels are assumed to occur with exponential transition rates depending on the current level. Due to the uniform activity levels, it is assumed that only transitions between neighboring states are possible. The result is a process where arrivals occur in multiples of a quantization step of A bits/pixel : $(0, A, 2A, ..., MA)$. The transition rates $r_{i,j}$ between state iA and jA is given by

$$r_{i,i+1} = (M-i)\alpha, \ \ i < M,$$

$$r_{i,i-1} = i\beta, \ \ i > 0,$$

$$r_{i,j} = 0 \ \text{elsewhere.}$$

The rate increment A and the transition rates α and β are chosen to match the mean, the variance and the auto covariance function of the experimental data. It is shown that this arrival process is equivalent with the aggregated rate from M independent mini sources, each alternating between transmitting 0 bits/pixel (off state) and A bits/pixel (on state) according to a Bernoulli distribution. When the information stream is packetized into cells, the bit rate (bits/pixel) becomes cell rate (cells/sec) and the model described above corresponds completely with the approximation for the superposition of sporadic sources described above.

3.4 Superposition of Variable Bit Rate Sources with multiple Activity Levels

In [20], Sen et al. generalize the results obtained in [16] to the superposition of video sources with multiple activity levels. The model includes both short-term and long-term correlations in the following way. The aggregate bit rate of the superposition changes among a finite number of fixed rate levels which are built up from two basic levels : a high rate A_h and a low rate A_l via integer combinations (i.e. a rate of $kA_h + mA_l$, where $0 \leq k \leq N_h$ and $0 \leq m \leq N_l$). It is show that this process can be seen as the aggregate arrival process of the superposition of N_h sporadic sources (with a bitrate of A_h in the active state) and the superposition of N_l sporadic sources (with a bitrate of A_l in the active state). From the first example we know that each of this superpositions is a D-BMAP, and hence, as the superposition of D-BMAP's is again a D-BMAP, the aggregate arrival process belongs to the class of D-BMAP's.

4 The D-BMAP/G/1/N Queue

Consider a discrete-time single server queue with capacity N. The input process is a D-BMAP characterized by the $m \times m$ matrices $\mathbf{D}_n$, $n \geq 0$. The service times are i.i.d. with general distribution, the z-transform of which is denoted by $G(z) = \sum_{k=1}^{\infty} g_k \, z^k$.

4.1 The embedded process

First we introduce the probability matrices that needed in the sequel.
Let $[\mathbf{A}_n^{(k)}]_{i,j}$ be the conditional probability that during the interval $(0,k]$ there are n arrivals and that at the end the phase of the arrival process is j, given that the process started at 0 in phase i. Then if we denote $\mathbf{D}(z) = \sum_{n=0}^{\infty} \mathbf{D}_n\ z^n$,

$$\mathbf{A}^{(k)}(z) = \sum_{n=0}^{\infty} \mathbf{A}_n^{(k)} z^n = [\mathbf{A}^{(1)}(z)]^k = [\mathbf{D}(z)]^k.$$

Let $[\mathbf{A}_n]_{i,j}$ be the probability that during a service there are n arrivals and that at the end the phase of the arrival process is j, given that the service started with the arrival process in phase i. Then

$$\mathbf{A}_n = \sum_{k=1}^{\infty} g_k \mathbf{A}_n^{(k)},$$

with the assumption that $g_0 = 0$. Let

$$\mathbf{A}(z) = \sum_{n=0}^{\infty} \mathbf{A}_n z^n \quad \text{and} \quad \mathbf{A} = \mathbf{A}(1).$$

Let furthermore $[\mathbf{B}_n]_{i,j}$ be the probability that, given a departure which leaves the system empty and the arrival process in phase i, at the next departure the arrival process is in phase j and there have been $n+1$ arrivals meanwhile. It is straightforward to show that

$$\mathbf{B}_n = (\mathbf{I} - \mathbf{D}_0)^{-1} \sum_{j=0}^{n} \mathbf{D}_{j+1} \mathbf{A}_{n-j}.$$

Let $\mathbf{B}(z) = \sum_{n=0}^{\infty} \mathbf{B}_n\ z^n$, then

$$\mathbf{B}(z) = z^{-1}\ (\mathbf{I} - \mathbf{D}_0)^{-1}\ [\mathbf{D}(z) - \mathbf{D}_0]\mathbf{A}(z),$$

and

$$\mathbf{B} = \mathbf{B}(1) = (\mathbf{I} - \mathbf{D}_0)^{-1}\ [\mathbf{D} - \mathbf{D}_0]\ \mathbf{A}.$$

We consider the system at departure epochs $t_0, t_1, t_2, \ldots$. Let $L(t_k)$ be the number of customers in the queue at instant t_k and let $J(t_k)$ be the phase of the arrival process at t_k. Then clearly $\{(L(t_k), J(t_k), t_{k+1} - t_k), k \geq 0\}$ is a semi-Markov process Q with state space $\{0, 1, \ldots, N-1\} \times \{1, \ldots, m\}$. Denote the joint probability distribution of the queue length and the phase of the arrival process at departures as the vector

$$\overline{\mathbf{x}} = \{\overline{\mathbf{x}}_0, \ldots, \overline{\mathbf{x}}_{N-1}\},$$

where

$$\overline{\mathbf{x}}_n = (x_{n,1}, ..., x_{n,m}), \qquad 0 \le n \le N-1,$$

with

$$x_{n,j} = \lim_{k\to\infty} \mathbf{P}\{L(t_k) = n, J(t_l) = j\}.$$

The vector $\overline{\mathbf{x}}$ is the invariant probability vector of the irreducible stochastic matrix $\mathbf{Q}$,

$$\mathbf{Q} = \begin{pmatrix} \mathbf{B}_0 & \mathbf{B}_1 & \mathbf{B}_2 & ... & \mathbf{B}_{N-2} & \sum_{n=N-1}^{\infty} \mathbf{B}_n \\ \mathbf{A}_0 & \mathbf{A}_1 & \mathbf{A}_2 & ... & \mathbf{A}_{N-2} & \sum_{n=N-1}^{\infty} \mathbf{A}_n \\ \mathbf{0} & \mathbf{A}_0 & \mathbf{A}_1 & ... & \mathbf{A}_{N-3} & \sum_{n=N-2}^{\infty} \mathbf{A}_n \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & ... & \mathbf{A}_0 & \sum_{n=1}^{\infty} \mathbf{A}_n \end{pmatrix}$$

The structure of $\mathbf{Q}$ shows that this queueing system has an embedded finite Markov chain of $M/G/1$-type (see also [1]).

4.2 The Queue Length Distribution at Departure Epochs

We give an efficient algorithm to determine the stationary probability distribution of this chain. A similar reasoning may be found in [10], [11] and [14].
We apply the following result (see e.g. [9]), also used by Grassmann, Taksar and Heyman [8] (extended to block partitioned matrices). Consider an $m \times m$ matrix of the form

$$\mathbf{X}_0 = \begin{pmatrix} x & \overline{b} \\ \overline{a}' & \mathbf{Y}_1 \end{pmatrix},$$

where $\overline{a}$ and $\overline{b}$ are $(m-1)$ dimensional vectors and where $\mathbf{Y}_1$ is an $(m-1)\times(m-1)$ matrix. Consider the matrix $\mathbf{X}_1$ which is obtained by adding to the transition probabilities from state i to j, $2 \le i, j \le m$, those transitions which go from i to j via state 1 :

$$\mathbf{X}_1 = \mathbf{Y}_1 + \overline{a}'\,(1-x)^{-1}\,\overline{b}.$$

The steady state probability vector of $\mathbf{X}_0$ can easily be computed from the one corresponding to $\mathbf{X}_1$. Applying this scheme several times, we obtain a sequence of matrices with decreasing dimension $\mathbf{X}_0, \mathbf{X}_1, \mathbf{X}_2, ...\mathbf{X}_m$. The last element $\mathbf{X}_m$ is a scalar. Now we can compute recursively the steady state vector of $\mathbf{X}_i$, $i = m-1, m-2, ..., 1$, and finally obtain the steady state vector of $\mathbf{X}_0$.

We apply this method, generalized to block partitioned matrices, to the above chain of M/G/1 type . For notational convenience, we let

$$\sum_{n=N-1}^{\infty} \mathbf{B}_n = \tilde{\mathbf{B}}_{N-1} \quad \text{and} \quad \sum_{n=k}^{\infty} \mathbf{A}_n = \tilde{\mathbf{A}}_k.$$

After the first step, and letting

$$\mathbf{C}_{1,i} = \mathbf{A}_{i+1} + \mathbf{A}_0(\mathbf{I} - \mathbf{B}_0)^{-1}\mathbf{B}_{i+1}, \ \ 0 \le i \le N-3$$

$$\tilde{\mathbf{C}}_{1,N-2} = \tilde{\mathbf{A}}_{N-1} + \mathbf{A}_0(\mathbf{I} - \mathbf{B}_0)^{-1}\tilde{\mathbf{B}}_{N-1},$$

we obtain the following block partitioned matrix (number of rows and columns is decreased by the dimension of a block)

$$\mathbf{Q}_1 = \begin{pmatrix} \mathbf{C}_{1,0} & \mathbf{C}_{1,1} & \mathbf{C}_{1,2} & \dots & \mathbf{C}_{1,N-3} & \tilde{\mathbf{C}}_{1,N-2} \\ \mathbf{A}_0 & \mathbf{A}_1 & \mathbf{A}_2 & \dots & \mathbf{A}_{N-3} & \tilde{\mathbf{A}}_{N-2} \\ \mathbf{0} & \mathbf{A}_0 & \mathbf{A}_1 & \dots & \mathbf{A}_{N-4} & \tilde{\mathbf{A}}_{N-3} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{A}_0 & \tilde{\mathbf{A}}_1 \end{pmatrix}.$$

Remark that the (j,k) entry of the matrix $\mathbf{C}_{1,i}$ is the probability, that given the process starts in state $(1,j)$, it enters a level higher than or equal to level 1 for the first time by hitting state $(i+1,k)$. A similar probabilistic interpretation is possible for the matrix $\tilde{\mathbf{C}}_{1,N-2}$. We immediately see that the resulting matrix $\mathbf{Q}_1$ is again of finite M/G/1 type. We apply the Grassman et al. result k times, we then obtain

$$\mathbf{Q}_k = \begin{pmatrix} \mathbf{C}_{k,0} & \mathbf{C}_{k,1} & \mathbf{C}_{k,2} & \dots & \mathbf{C}_{1,N-k-2} & \tilde{\mathbf{C}}_{1,N-k-1} \\ \mathbf{A}_0 & \mathbf{A}_1 & \mathbf{A}_2 & \dots & \mathbf{A}_{N-k-2} & \tilde{\mathbf{A}}_{N-k-1} \\ \mathbf{0} & \mathbf{A}_0 & \mathbf{A}_1 & \dots & \mathbf{A}_{N-k-3} & \tilde{\mathbf{A}}_{N-k-2} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{A}_0 & \tilde{\mathbf{A}}_1 \end{pmatrix},$$

with

$$\mathbf{C}_{k,i} = \mathbf{A}_{i+1} + \mathbf{A}_0(\mathbf{I} - \mathbf{C}_{k-1,0})^{-1}\mathbf{C}_{k-1,i+1}, \ \ 0 \le i \le N-k-2, \ \ 2 \le k \le N-2$$

$$\tilde{\mathbf{C}}_{k,N-k-1} = \tilde{\mathbf{A}}_{N-k} + \mathbf{A}_0(\mathbf{I} - \mathbf{C}_{k-1,0})^{-1}\tilde{\mathbf{C}}_{k-1,N-k}.$$

Finally we obtain the matrix

$$\mathbf{Q}_{N-1} = \tilde{\mathbf{A}}_1 + \mathbf{A}_0(\mathbf{I} - \mathbf{C}_{N-2,0})^{-1}\tilde{\mathbf{C}}_{N-2,1}.$$

Let the vector $\overline{\mathbf{x}}_{N-1}$ satisfy

$$\overline{\mathbf{x}}_{N-1}\,\mathbf{Q}_{N-1} = \overline{\mathbf{x}}_{N-1}.$$

From this we can compute the steady state vector $(\overline{\mathbf{x}}_{N-1}\ \overline{\mathbf{x}}_{N-2})$ of $\mathbf{Q}_{N-2}$ using

$$\overline{\mathbf{x}}_{N-2} = \overline{\mathbf{x}}_{N-1}\,\mathbf{A}_0(\mathbf{I} - \mathbf{C}_{N-2,0})^{-1}.$$

In general, the steady state vector $\overline{\mathbf{x}} = (\overline{\mathbf{x}}_0, \overline{\mathbf{x}}_1, ..., \overline{\mathbf{x}}_{N-1})$ can be computed using the following recursive formulas

$$\overline{\mathbf{x}}_{N-1}[\tilde{\mathbf{A}}_1 + \mathbf{A}_0(\mathbf{I} - \mathbf{C}_{N-2,0})^{-1}\tilde{\mathbf{C}}_{N-2,1}] = \overline{\mathbf{x}}_{N-1},$$

$$\overline{\mathbf{x}}_k = \overline{\mathbf{x}}_{k+1}\mathbf{A}_0(\mathbf{I} - \mathbf{C}_{k,0})^{-1}, \quad 1 \le k \le N-2,$$

$$\overline{\mathbf{x}}_0 = \overline{\mathbf{x}}_1\mathbf{A}_0(\mathbf{I} - \mathbf{B}_0)^{-1}.$$

The complexity of this algorithm is of the order M^3N^2, where M is the dimension of a block and where N is the number of rows of blocks. The required memory is 2 M^2N. Indeed, it is sufficient to store the first two rows

$$\begin{pmatrix} \mathbf{B}_0 & \mathbf{B}_1 & \mathbf{B}_2 & ... & \mathbf{B}_{N-2} & \tilde{\mathbf{B}}_{N-1} \\ \mathbf{A}_0 & \mathbf{A}_1 & \mathbf{A}_2 & ... & \mathbf{A}_{N-2} & \tilde{\mathbf{A}}_{N-1} \end{pmatrix}.$$

Each time the Grassman et al. method is applied we use the first row to store the matrices $\mathbf{C}_{k,i}$ and $\tilde{\mathbf{C}}_{k,N-k-1}$ (since the matrices $\mathbf{B}_k$, $k = 1, ..., N-2$ and $\tilde{\mathbf{B}}_{N-1}$ are not used after the first step). Moreover the matrix $\mathbf{A}_0(\mathbf{I} - \mathbf{C}_{k,0})^{-1}$ is also stored in the first row, and can be used when evaluating the vectors $\overline{\mathbf{x}}_k$.

4.3 The Queue Length Distribution at an Arbitrary Time Instant

Define the joint probability distribution of the queue length and the arrival phase at an arbitrary time instant $t \in N$ by

$$y(n, j; t \mid n_0, j_0) = \mathbf{P}\{L(t) = n, J(t) = j \mid L(0) = n_0, J(0) = j_0\},$$

and let

$$y(n,j) = \lim_{t\to\infty} y(n,j;t \mid n_0, j_0)$$

$$\overline{\mathbf{y}}_n = (y(n,1), \ldots, y(n,m)).$$

Before giving explicit expressions for the vectors $\overline{\mathbf{y}}_n$ we need the fundamental mean E^* of the semi-Markov process Q, i.e. the average time between an arbitrary departure and the next departure.

Lemma 1 *The fundamental mean E^* of the semi-Markov process Q is given by*

$$E^* = \mathbf{E}[G] \; + \; \overline{\mathbf{x}}_0(\mathbf{I}-\mathbf{D}_0)^{-1}\,\overline{\mathbf{e}}.$$

Proof. Since the fundamental mean E^* is the average time between an arbitrary departure from the queue and the next departure, we have

$$\begin{aligned}
E^* &= \overline{\mathbf{x}}_0[\sum_{n=1}^{\infty}\sum_{l=0}^{\infty}(l+1)\,(\mathbf{D}_0)^l\,\mathbf{D}_n \; + \; \mathbf{E}[G]\,]\,\overline{\mathbf{e}} + \sum_{n=1}^{N-1}\overline{\mathbf{x}}_n\,\overline{\mathbf{e}}\,\mathbf{E}[G] \\
&= \mathbf{E}[G] \; + \; \overline{\mathbf{x}}_0\sum_{l=0}^{\infty}(l+1)\,(\mathbf{D}_0)^l\,(\mathbf{D}-\mathbf{D}_0)\,\overline{\mathbf{e}} \\
&= \mathbf{E}[G] \; + \; \overline{\mathbf{x}}_0(\mathbf{I}-\mathbf{D}_0)^{-2}\,(\mathbf{D}-\mathbf{D}_0)\,\overline{\mathbf{e}} \\
&= \mathbf{E}[G] \; + \; \overline{\mathbf{x}}_0(\mathbf{I}-\mathbf{D}_0)^{-1}\,\overline{\mathbf{e}}. \quad \blacksquare
\end{aligned}$$

Reamrk that a direct argument on the length of the idle period of this system leads to the same result.
The next theorem gives expressions for the vectors $\overline{\mathbf{y}}_n$ suitable for direct computation.

Theorem 1 *The vectors $\overline{\mathbf{y}}_n$ are given by*

$$\overline{\mathbf{y}}_0 = \frac{1}{E^*}\,\overline{\mathbf{x}}_0\,(\mathbf{I}-\mathbf{D}_0)^{-1}$$

$$\overline{\mathbf{y}}_{n+1} = [\sum_{i=0}^{n}\overline{\mathbf{y}}_i\mathbf{D}_{n+1-i} + \frac{1}{E^*}\,(\overline{\mathbf{x}}_n - \overline{\mathbf{x}}_{n+1})](\mathbf{I}-\mathbf{D}_0)^{-1}, \quad 0 \le n < N-1,$$

$$\overline{\mathbf{y}}_N = \overline{\boldsymbol{\pi}} - \sum_{n=0}^{N-1}\overline{\mathbf{y}}_n.$$

Proof. Define $[d\mathbf{M}(u)]_{n,j}$ to be the elementary probability that at the end of the u-th time slot the semi-Markov process Q enters the state (n, j).
First we compute $\overline{\mathbf{y}}_0$. Clearly,

$$\overline{\mathbf{y}}_0 = \lim_{t \to \infty} \sum_{u=1}^{t} d\mathbf{M}_0(u) \, (\mathbf{D}_0)^{t-u}.$$

Applying the key renewal theorem results into

$$\overline{\mathbf{y}}_0 = \frac{1}{E^*} \, \overline{\mathbf{x}}_0 \, (\mathbf{I} - \mathbf{D}_0)^{-1}.$$

In what follows we first compute $\overline{\mathbf{y}}_n$, $n \geq 1$, for the infinite system. Then, since the vectors $\overline{\mathbf{y}}_n$ satisfy the same equations for both the finite and the infinite system, it is easy to obtain the required results.
First we compute the joint probability of the number of customers in the system and the phase of the arrival process at an arbitrary instant and the time until the next service initiation.
Define $[\overline{\mathbf{r}}(t; n, k)]_j$ to be the joint probability that at time t there are n customers in the system, that the arrival process is in phase j and that the next service initiation occurs no later than time $t + k$, with $t, n, k \in N$, and $k, n \geq 1$. Furthermore, define

$$\overline{\mathbf{r}}(n, k) = \lim_{t \to \infty} \overline{\mathbf{r}}(t; n, k),$$

and denote the transform vectors

$$\overline{\mathbf{r}}^*(z, y) = \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \overline{\mathbf{r}}(n, k) y^k z^n.$$

Using the law of total probability, it possible to decompose $\overline{\mathbf{r}}^*(z, y)$ into two vectors $\overline{\mathbf{r}}_i^*(z, y)$, $i = 1, 2$, according to the following two cases :

(i) the arbitrary time instant falls during the first service of a busy period

(ii) the arbitrary time instant falls during the second or later service of a busy period.

Let us first compute $\overline{\mathbf{r}}_1^*(z, y)$.
Suppose that at instant u, $0 \leq u \leq t$ a customer leaves the system empty and that the idle period ends after l time units, $0 \leq l \leq t - u$, with the arrival of i customers, $1 \leq i \leq n$. Then

$$\overline{\mathbf{r}}_1(t; n, k) = \sum_{u=0}^{t} \sum_{l=0}^{t-u} \sum_{i=1}^{n} d\mathbf{M}_0(u) \, (\mathbf{D}_0)^l \, (\mathbf{D}_i) \, \mathbf{A}_{n-i}^{(t-u-l)} \, g_{t+k-u-l}.$$

Let $t \to \infty$ and apply a discrete version of the key renewal theorem, together with a change of variables, then

$$\bar{\mathbf{r}}_1(n,k) = \sum_{u=0}^{\infty} \sum_{l=0}^{u} \sum_{i=1}^{n} \frac{1}{E^*} \bar{\mathbf{x}}_0 \, (\mathbf{D}_0)^l \, (\mathbf{D}_i) \, \mathbf{A}_{n-i}^{(u-l)} g_{u+k-l}.$$

Taking transforms yields

$$\bar{\mathbf{r}}_1^*(n,y) = \frac{1}{E^*} \bar{\mathbf{x}}_0 \sum_{u=0}^{\infty} \sum_{l=0}^{\infty} \sum_{k=1}^{\infty} \sum_{i=1}^{n} (\mathbf{D}_0)^l \, (\mathbf{D}_i) \, \mathbf{A}_{n-i}^{u} \, g_{u+k} \, y^k.$$

Take the z-transform and apply the product rule for z-transforms, then

$$\bar{\mathbf{r}}_1^*(z,y) = \frac{1}{E^*} \bar{\mathbf{x}}_0 \, (\mathbf{I} - \mathbf{D}_0)^{-1} \sum_{u=0}^{\infty} \sum_{k=1}^{\infty} [\mathbf{D}(z) - \mathbf{D}_0] \mathbf{A}^u(z) \, g_{u+k} \;\; y^k.$$

It is easy to show that,

$$\bar{\mathbf{r}}_1^*(z,y) = \frac{1}{E^*} \bar{\mathbf{x}}_0 \, (\mathbf{I} - \mathbf{D}_0)^{-1} \, [\mathbf{D}(z) - \mathbf{D}_0][G(y)\mathbf{I} - \mathbf{A}(z)] \, [y\mathbf{I} - \mathbf{D}(z)]^{-1} \, y.$$

Let us now compute $\bar{\mathbf{r}}_2(t;n,k)$. From

$$\bar{\mathbf{r}}_2(t;n,k) = \sum_{u=0}^{t} \sum_{i=1}^{n} d\mathbf{M}_i(u) \, \mathbf{A}_{n-i}^{(t-u)} \, g_{t+k-u}.$$

and after manipulations similar to those in the previous case we obtain

$$\bar{\mathbf{r}}_2^*(z,y) = \frac{1}{E^*} \, (\overline{\mathbf{X}_0} - \bar{\mathbf{x}}_0) \, [G(y)\mathbf{I} - \mathbf{A}(z)] \, [y\mathbf{I} - \mathbf{D}(z)]^{-1} \, y.$$

Denote for the infinite system $\overline{\mathbf{X}}(z)$, resp. $\overline{\mathbf{Y}}(z)$, the generating function of the queue length distribution at departures, resp. at an arbitrary instant. Then from the definition of the vectors $\bar{\mathbf{r}}_i^*(z,y)$, it follows that

$$\overline{\mathbf{Y}}(z) = \bar{\mathbf{y}}_0 + \bar{\mathbf{r}}_1^*(z,1) + \bar{\mathbf{r}}_2^*(z,1).$$

Using the identity

$$\overline{\mathbf{X}}(z)[z\mathbf{I} - \mathbf{A}(z)] = \bar{\mathbf{x}}_0 \, [z\mathbf{B}(z) - \mathbf{A}(z)],$$

together with

$$\mathbf{B}(z) = z^{-1} \, (\mathbf{I} - \mathbf{D}_0)^{-1} \, [\mathbf{D}(z) - \mathbf{D}_0]\mathbf{A}(z),$$

results into

$$\overline{\mathbf{Y}}(z) = \frac{1}{E^*} \, (z-1)\overline{\mathbf{X}}(z) \, (\mathbf{I} - \mathbf{D}(z))^{-1}.$$

Hence,

$$\overline{\mathbf{Y}}(z)(\mathbf{I}-\mathbf{D}_0) = z\,\overline{\mathbf{Y}}(z)\,\mathbf{D}(z) \;+\; \frac{1}{E^*}\,(z-1)\,\overline{\mathbf{X}}(z),$$

from which we immediately derive the required result. Since the state equations for $\overline{\mathbf{y}}_n$, $0 \leq n \leq N-1$ are the same for both the finite and the infinite system, we obtain that

$$\overline{\mathbf{y}}_0 = \frac{1}{E^*}\,\overline{\mathbf{x}}_0\,(\mathbf{I}-\mathbf{D}_0)^{-1}$$

$$\overline{\mathbf{y}}_{n+1} = [\sum_{i=0}^{n}\overline{\mathbf{y}}_i\mathbf{D}_{n+1-i} + \frac{1}{E^*}\,(\overline{\mathbf{x}}_n - \overline{\mathbf{x}}_{n+1})](\mathbf{I}-\mathbf{D}_0)^{-1}, \quad 0 \leq n < N-1,$$

$$\overline{\mathbf{y}}_N = \overline{\boldsymbol{\pi}} - \sum_{n=0}^{N-1}\overline{\mathbf{y}}_n. \quad \blacksquare$$

From this state probabilities it is now possible to derive the probability of cell loss. This loss probability is obtained form the mean number of arrivals and the mean number of arrivals that are lost per time unit. Hence,

$$P_b = \frac{1}{\overline{\boldsymbol{\pi}}\,(\sum_{k=1}^{\infty} k\mathbf{D}_k)\,\overline{\mathbf{e}}}\,[\sum_{n=0}^{N}\sum_{k=1}^{\infty}(k-N+n)^+\,\overline{\mathbf{y}}_n\mathbf{D}_k\overline{\mathbf{e}}],$$

where $(k-N+n)^+ = \max(0, k-N+n)$.

4.4 The Output Process of the D-BMAP/D/1/N Queue

In what follows, we show that the output process of the finite capacity D-BMAP/D/1 queue is a D-MAP.

In fact, the result we prove is somewhat stronger and can be formulated as follows :

Theorem 2 *The output process of a slotted Markovian queueing system with a upper block-Hessenberg transition matrix is a D-MAP.*

Proof. Consider a finite capacity single server queue. The service time is supposed to be constant and is chosen as time unit. The server operates as a slotted system (i.e. when the system is empty, an arriving customer has to wait for the next time slot before being served). Observe this system at the end of each time slot and suppose that its queue length can be described by a two dimensional Markov chain with an upper block-Hessenberg transition matrix :

$$\mathbf{Q} = \begin{pmatrix} \mathbf{B}_{0,0} & \mathbf{B}_{0,1} & \mathbf{B}_{0,2} & \dots & \mathbf{B}_{0,N-1} & \mathbf{B}_{0,N} \\ \mathbf{B}_{1,0} & \mathbf{B}_{1,1} & \mathbf{B}_{1,2} & \dots & \mathbf{B}_{1,N-1} & \mathbf{B}_{1,N} \\ \mathbf{0} & \mathbf{B}_{2,0} & \mathbf{B}_{2,1} & \dots & \mathbf{B}_{2,N-2} & \mathbf{B}_{2,N-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{B}_{N,0} & \mathbf{B}_{N,1} \end{pmatrix}.$$

Remark that the D-BMAP/D/1/N queue has such a transition matrix. The output process of this system is a D-MAP with parameters

$$\mathbf{D}_0 = \begin{pmatrix} \mathbf{B}_{0,0} & \mathbf{B}_{0,1} & \mathbf{B}_{0,2} & \dots & \mathbf{B}_{0,N-1} & \mathbf{B}_{0,N} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \end{pmatrix},$$

$$\mathbf{D}_1 = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{B}_{1,0} & \mathbf{B}_{1,1} & \mathbf{B}_{1,2} & \dots & \mathbf{B}_{1,N-1} & \mathbf{B}_{1,N} \\ \mathbf{0} & \mathbf{B}_{2,0} & \mathbf{B}_{2,1} & \dots & \mathbf{B}_{2,N-2} & \mathbf{B}_{2,N-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{B}_{N,0} & \mathbf{B}_{N,1} \end{pmatrix}. \blacksquare$$

From this theorem it follows that the D-BMAP can be used as a generic component for traffic studies in integrated communication networks.

5 Conclusion

In this contribution, we have introduced a versatile class of discrete-time batch Markovian arrival processes (D-BMAP's), which can be used as basic model for analytical performance evaluation problems in integrated communication networks. We have shown that a number of known models for traffic sources fit in the framework of our D-BMAP. Among other properties, we have proved that the single source DMAP model ([2]) together with the superposition of DMAP's, is a special case of the D-BMAP. Furthermore, the D-BMAP can be used as approximation for the superposition of sporadic sources and also for the superposition of video sources, both with uniform and multiple activity scenes. A matrix-analytical approach of the D-BMAP/G/1/N queue leads to simple algorithms for

computing many performance measures of interest.
The results obtained in [16] and [20] seem to be very promising for a D-BMAP approximation of the superposition of video sources. Furthermore, we intend to investigate more elaborated models of multiplexers, which allow to evaluate the probability a tagged source loses a string of cells (see [4], [5]).

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