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Credit grace periods for one-time-only sales

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Abstract

Periods of sluggish demand and/or poor sales forecasts often leads to build-ups of unwanted inventories. When this happens, funds are tied up unnecessarily in unproductive endeavours and expected sales revenues are not realized. To counter such occurrences, vendors often retort to payment reduction schemes designed to encourage sales. The most common is a temporary purchase price reduction until inventories reach more acceptable levels. A lesser known but also quite popular strategy is to give the buyer a grace period within which the buyer is not required to pay for the merchandise already purchased. The evaluation of such strategy, from the buyer's perspective, is the purpose of this paper.

Keywords : Marketing/Finance, Inventory Theory, Pricing

1. INTRODUCTION

When faced with low sales and increasing inventory on hand, a vendor's attempt to overcome sluggish demand often includes the offer of either a sales price discount (PD) or a one-time opportunity to delay the payment (DOP) of the merchandise for a specific period of time. Whereas the methodology for the buyer's economic evaluation of the PD offer is well known (e.g. [4]-[6],[10],[11]), the same cannot be said for the DOP case. Hence, when comparing the two payment reduction modes, a common tendency is to search for the DOP policy which yields equivalent savings to those arising from the reduction in the unit purchase cost [7]. However, such tendency has its pitfalls. It is the purpose of this paper to outline the main differences between the DOP and the PD options, as it affects the buyer's purchasing decision, and to provide the appropriate methodology for the DOP case. To that effect, the next section outlines the nature of the DOP alternative via a numerical example. This is followed by a description of the steps needed to evaluate the DOP option. Numerical examples are included next to highlight the sensitivity of the optimal order quantity, with respect to fluctuations in the parameters. A Conclusions section completes the paper.

2. THE DELAY OF PAYMENT STRATEGY

2.1 Nature of the DOP Strategy.

A price discount provides the buyer with (i) lower purchasing costs; (ii) lower ordering costs, reflecting the decrease in the number of the now larger-size replenishments required to satisfy a given demand rate; (iii) lower financing costs, since the short-term financing of the purchase is usually set as a percentage of the per-unit purchasing price. In contrast, the DOP results in (i) no savings in purchasing costs, since the buyer does not receive a discount; (ii) lower ordering costs, as in the PD case; (iii) savings in financial costs for the duration of the grace period; and (iv) no savings in the physical carrying costs (insurance, storage and the like), since the buyer takes immediate possession of the merchandise.

The case for a unique DOP strategy rests upon the basic argument that, unlike a price discount, where the benefit of the per-unit discount is independent of the quantity bought, the per-unit benefit of the delayed payment is not constant. In other words, the benefit of credit does not increase proportionately with the additional purchase. As a result, it is not possible to find a per-unit price discount which is simultaneously (i) equivalent to some DOP offer; and (ii) constant and hence independent of the magnitude of the extraordinary purchase. Further elaboration upon this point appears in [3]-[5] and [8].

2.2 A Numerical Illustration.

The following numerical example may be used to illustrate the case outlined above. Suppose that the current order quantity, for convenience assumed to be the economic order quantity [9] or EOQ (the optimal order size for the standard model without either the price discount or the delay of payment options), is $Q = 1000$ units a month @ \$1 per unit. Assume further that the vendor is offering a credit of two months for an order of size $q > Q$. Consider two possibilities, namely two extraordinary orders whose sizes exceed Q by 2000 and 4000 units, respectively. Table 1 depicts the cash flows associated with all three alternatives. For comparison purposes, all evaluations are done at a time commencing with the reorder point.

[PLEASE INSERT TABLE 1 ABOUT HERE]

Under regular ordering, a constant outflow of \$1000 occurs at the end of every month, starting at time zero. On the other hand, for a special order of $q = 3000$ units, enough to cover an extra two months worth of consumption, no payment is required until the end of the second month. At this point, payment must be made not only of the regular $1000 purchase, but also of the extra two-month order, worth an additional $2000. From then onwards, a regular payment of $1000 is to take place at the end of each month. In addition, if the extraordinary purchase is increase to q = 5000 units, the entire $5000 payment is due at the end of the second month. This is so, even though sufficient inventory remains on hand to cover the consumption of months 3 and 4. Hence, no payments are due until the end of the fifth month, at which point the regular $1000 monthly disbursements are scheduled to resume.

In short, even a cursory look at the two DOP strategies listed in Table 1 suggests that the benefits from the credit period not only do not increase proportionately with the special order, but in fact decrease after a certain level. This clearly shows that attempts to capture the benefits of the credit period using the price discount method will result in misleading conclusions. Furthermore, the examples of Table 1 illustrate the main advantage to the purchaser, when a vendor allows a one-time-only opportunity for delaying payment. Revenues earned during the grace period which would otherwise be used immediately to finance the purchase of merchandise may now be earmarked to generate investment income for the duration of the extended payment period.

3. METHODOLOGY FOR THE DOP STRATEGY

3.1 Notation.

To evaluate the DOP strategy, the following notation is used. Let **s** be the cost of placing an order; **t**, the length of the DOP period; **d**, the purchase price per unit; **k**, the rate of return, per unit per year, on the funds invested, expressed as a percentage of d; i, the cost of borrowing, per unit per year, expressed as a percentage of d; **r**, the yearly demand rate; **q**, the number of units acquired in the extraordinary purchase; $\mathbf{h_1}$, the inventory carrying costs, other than financial, per unit per year, during the length of the DOP period, t, expressed as a percentage of d; $\mathbf{h_2=h_1+i}$, the inventory carrying costs, including financial, per unit per year, outside the DOP period, expressed as a percentage of d.

3.2 Optimization Objective.

Two cases need to be considered on the basis of the relationship between the length of the DOP period, t, and the length of the extraordinary replenishment cycle, q/r. For each case, Table 2 lists the expressions related to (i) the cash flows associated with the acceptance, Z(q), and the rejection, Z'(q), of the DOP strategy; and (ii) the optimal order sizes per cycle, q*, i.e. that which maximizes the difference between the cash flows, Z(q)-Z'(q).

[PLEASE INSERT TABLE 2 ABOUT HERE]

In turn, Z(q) may be expressed as the difference between two sets of cash flows.The first consists of the inflows arising out of the funds earmarked for inventory financing once the grace period is over. The second include the standard outflows of the basic EOQ model, namely those associated with the ordering or replacement costs and with the holding costs, be them financial or physical. In addition, the expression for Z'(q) represents the optimal EOQ cost [9].

3.3 Mathematical Development of Z(q).

The rationale for the mathematical expressions of the cash flows is as follows. First, regardless of the case, only one order is to be placed if the DOP offer is accepted. Hence, under either case, (i) the fixed replacement costs are given by s; and (ii) the physical carrying costs are charged at the rate of $\$dh_1$ per unit per year, for the time, q/r, it takes to deplete the inventory, on an average of q/2 units, for a total of $q^2dh_1/2r$.

Second, financial flows are of two types, namely those related with the financing of the inventory and those associated with the revenue cash inflows. Allocation of these flows are made on the basis of the following rules: (i) borrow only the amount needed and only when needed; (ii) if there is no need for borrowing, invest the revenue inflows for a return of \$kd per unit per year; and (iii) once borrowing takes place, use the sales cash inflows to repay the borrowed amount.

These rules lead to the appropriate expressions listed in Table 2. For, the financing outflows, the differences between the two cases arise out of the need or lack thereof to finance the q units comprising the extraordinary purchase, during the q/r time period. In Case 2, since payment is due after the DOP stock has already been depleted, no inventory financing flows occur during the q/r period. In Case 1, financing costs are incurred at the rate of \$id per unit per year, for an average of (q-rt)/2 units. Such costs are incurred during the (q/r - t) time period, starting at the end of the DOP grace period, t, and the end of the DOP stock, q/r. The end result is a total cash outflow of $\$id(q-rt)(q-rt)/2=\$id(q-rt)^2/2$.

Inflows arise out of interest, \$kd per unit, from sales revenue invested during the grace period which would otherwise have been used to finance the purchase of the q units. In Case 1, the interest accumulates at the rate of \$kd per unit on an average sales volume of rt/2 units during the grace period of length t, for a total of $\$rkdt^2/2$. In Case 2, the accumulation occurs in two periods. One covers the entire q/r interval for an inflow of \$kd per unit on an average of q/2 units, yielding an inflow of $\$q^2kd/2r$. The other includes the additional (t-q/r) time period where the sales revenue base covers the entire q units already sold, for an inflow of \$kd per unit and a total of \$(t-q/r)qkd.

Then combining the outflows and inflows yields the following expression for Z(q):

$$Z(q) = \begin{cases} rkdt^2/2 - s - q^2dh_1/2r - (q-rt)^2id/2r, & \text{if } t \le q/r \\ q^2kd/2r + (t-q/r)qkd - s - q^2dh_1/2r, & \text{if } t > q/r \end{cases} \tag{1}$$

3.4 Optimality Results.

The derivation of the optimality conditions follows standard optimization rules. Setting the first derivative of Z(q)-Z'(q) with respect to q equal to zero yields the expression for q^* given in Table 2 for both cases. Further, substituting its optimal value for q in the definition of the cases and rearranging terms yields the optimal constraints on the grace period, t. These constraints provide a rather intuitively appealing marginal-cost/marginal-savings economic interpretation to the distinction between the two cases. Whether or not the grace period, t, exceeds the optimal DOP order depletion time, q^*/r, depends upon the magnitude of the physical carrying costs of holding inventory during t, relative to the holding costs of the alternate, non-DOP option.

Note that the difference in q^* between the two cases arises out of the difference between the rate of borrowing, i, and the rate of return on investment, k. When both rates are equal, i.e. $i = k = h_2-h_1$, then so are the expressions for their respective optimal order quantities, obviating the need to distinguish between the two cases. If, in addition, the financial costs dominate over the physical carrying costs, as it is often the case in practice, then $h_1 \approx 0$. As a consequence, when $h_1=0$ and i=k, the optimal ordering quantity would equal $q^* = rt+Q$. This is as it should be.

The privilege of credit yields maximum benefit if the additional quantity ordered just equals the demand during the DOP period.

Further, note that q* is independent of the rate of return on the funds invested, k, for Case 1. This reflects the fact that the opportunity to invest is restricted to a grace period, t, below q*/r, irrespective of the magnitude of the special order. On the other hand, an examination of the first two derivatives of q* with respect to k, suggests that, with the restriction lifted in Case 2, q* (i) increases with k, because of the larger and earlier availability of funds for investment; (ii) at a decreasing rate, since the time, (t-q*/r), available for investment decreases.

With respect to the effect of the borrowing rate, i, the following observations may be made. First, the DOP profit function, Z(q), in Case 2 is independent of i, because there are no financial costs during the entire q/r depletion time, as the grace period extends beyond q/r. It is only through the non-DOP alternative that i affects Case 2's decision. On the other hand, in Case 1, financing costs do occur between the end of the grace period and that of the depletion time, q/r, of the special order, without any counterbalancing investment inflows during the said (t, q/r) period. Second, these different influences are also reflected on the effect of i on the optimal DOP order quantity, q*. As shown in Table 2, when t>q*/r, the impact of i on q* occurs through $h_2Q=(h_1+i)Q$. Once again, an examination of the first two derivatives of h_2Q and q* with respect to i reveals that as the holding costs of the non-DOP alternative increase, (i)so does q*, since q* becomes more attractive, the higher the costs of the alternate option; but (ii) at a decreasing rate, due to the diminishing length of the inflow (t,q*/r) period. Once q* exceeds rt in Case 1, increases in the borrowing rate result in lower values for Q and, hence, for q*. Furthermore, these decreases occur at an increasing rate to avoid the extra financing outflows incurred without the compensating additional inflows obtained in Case 2.

4. A NUMERICAL EXAMPLE

For the example of this section, the following values of the parameters are used, unless otherwise stated: s=25$/order; t=1/12 or one month; d=2.5$/unit; k=18%; r=36000 units/year; i=15%; and h_1=10%. Then, Q=1604 units, q*=3532 units and, thus, q*-Q=1928 extra units are placed in the special purchase. Of more interest, for the purposes of this paper, is to study the sensitivity of fluctuations in the parameters on the magnitude of q*.

Figure 1 depicts graphically the effect on q* of changes in both financial, i, and non-financial, h_1, carrying costs. Observe that for small values of h_1, $q^* \rightarrow Q$. Hence, q* is a decreasing function of i, reflecting Case 1's characteristics discussed above. However, as h_1 increases along with the relative importance of the non-financial with respect to the financial carrying costs, so does q*-Q and at a rate, in absolute terms, higher than the corresponding rate of decrease of Q. At some point (for this example, in the h_1=0.06 range, according to Figure 1), the rt>q* of Case 2 is valid. Then, q* increases with i, as the DOP alternative becomes more and more attractive.

[PLEASE INSERT FIGURE 1 ABOUT HERE]

Similar analysis can be carried out with respect to the other parameters. However, it can readily be seen that their effect on q* is in the same direction as for the regular order quantity. Thus, increases in the demand rate, r, or in the replenishment cost, s, or decreases in the purchase price, d, lead to larger orders, as would longer grace periods, t.

5. CONCLUSIONS

Even though it is common practice to convert any payment reduction scheme into its price-discount equivalent, this paper shows that such an action is not needed, at least as far as the DOP option is concerned. This is due mainly to the fact that (i) there is not a one-to-one correspondence between the DOP and the PD options; and that (ii) the proposed methodology to evaluate the DOP alternative indicates the ease with which a DOP order quantity may be estimated directly. As a result, implementation problems of DOP policies can be kept at a minimum, with the corresponding enhancement of its usefulness in practice.

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TABLE 1 Payment Pattern for Various Order Sizes							
		End of the Month Cash Outflows					
Order Size q	Extra Units Purchased q-Q	0	1	2	3	4	5
1000	0	1000	1000	1000	1000	1000	1000
3000	2000	0	0	3000	1000	1000	1000
5000	4000	0	0	5000	0	0	1000

TABLE 2 Cash Flow Patterns and Optimal Order Quantities		
	Case 1: $t \leq q/r$	*Case 2: $t>q/r$*
A. Cash Flows, $Z(q)$, if the DOP option is accepted		
- Fixed Replacement Costs	s	s
- Physical Carrying Costs	$q^2dh_1/2r$	$q^2dh_1/2r$
- Financing Costs	$(q-rt)^2id/2r$	0
+ Additional Income on Funds Invested: During q/r	$rkdt^2/2$	$q^2kd/2r$
Beyond q/r	0	$(t-q/r)qkd$
B. Cash Flows, $Z'(q)$, if the DOP option is rejected[1]	$qQdh_2 / r$	
C. Optimal Order Quantity[1]**, q^***	$Q + rti/h_2$	$(h_2Q+rtk)/(h_1+k)$
D. Optimal constraint on t	$rth_1 \leq Qh_2$	$rth_1 \geq Qh_2$

[1] **Note**: $Q = (2rs/dh_2)^{1/2}$ is the Economic Order Quantity.

Figure 1

Effect of Carrying Costs, h_1 and i, on the Special Order Quantity