A TUTORIAL ON PRODUCTION MODELS [1)]

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1. *Introduction*

Production models have been designed for *planning* and *control* purposes in a *production system*. Despite the obvious difference in meaning of the words "planning" and "control", a good deal of confusion has arisen about their use in production. We will regard production planning as dealing entirely with *pre-production* activities (e.g. demand forecasting, planning of aggregate production, etc...). Production *control* we consider to be essentially a *during-production* activity, i.e. an activity to ensure that products are manufactured according to the previously determined production plan (e.g. recording job progress, modification of original targets, etc...).

Although production models have mainly been presented as *pre-production devices*, one should keep in mind - especially during the design phase - that planning and control are only components of the same overall problem.

Production models may be built for different types of production systems. Generally one distinguishes :

- large scale, one-time systems or project systems
- continuous or flow-shop systems
- intermittent or job-shop systems.

The objective function of a production model will be a cost minimization function, the specific terms of which depend on the planning horizon. Generally one distinguishes three planning levels, namely

a) long range planning, which involves the major adjustments of plant capacity to match projected demands;

b) medium range planning or aggregate planning, which include hiring, firing and overtime decisions;

c) detailed scheduling, including machine-worker assignments and sequencing decisions.

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Clearly long range, medium range and short range scheduling problems are only components of the same overall production planning problem. However, the most standard strategy for solving large and complex problems has always been to break it down into smaller parts which are more manageable. As a consequence, the independent problems of long range, medium range and short range planning have achieved *as such* some measure of acceptance by the business community.

2. The Aggregate Planning Problem

We will mainly direct our attention to the medium range (or aggregate) planning problem, the objective of which is to ensure smooth production, and to avoid unnecessary fluctuations in the size of workforce, overtime, etc... Aggregate planning is concerned with the setting of production rates and workforce levels and, hence, the determination of the finished inventory levels and overtime or sub-contracting requirements necessary to satisfy a given (fluctuating) demand pattern.

The following costs are important :

1. production costs : the cost of production of a particular item at a given rate
2. stockholding costs
3. costs associated to changes in production rates (capacity costs)
4. shortage costs.

It is easy to understand why aggregate planning attracted so much attention. Medium term decisions determine the operating conditions, or framework, within which the short range decisions are made. Short term models typically consider the assignment of, or rather the sequencing of, jobs on machines. Typically in a job shop these two models are available. The detailed model is generally computationally too costly to be used alone and unguided for the period by period production and workforce decisions of intermediate planning. Conversely, the computational cost of making aggregate decisions is generally low. But the aggregate model, because it suppresses so much detail, cannot take detail account of the period by period conditions.

If computation were sufficiently cheap and fast, a decision maker could evaluate all interesting sets of production level - workforce decision, over the decision horizon, and select the best solution. As an example, let us consider an aggregate decision maker who has to determine two decision variables over a 10-period horizon. Suppose moreover that each of these 20 (2 x 10) variables is restricted to 10 possible discrete values. A complete enumeration will require the evaluation of 10^{20} possible solutions. Suppose that a computer requires on the average 10^{-3} seconds to make one evaluation of the objective function. At this speed, total enumeration would require $10^{-3} . 10^{20} = 10^{17}$ seconds. This compares to the average human life-span of $2{,}2 \times 10^{9}$ seconds.

As a consequence, a wealth of models has been presented in the aggregate-planning literature. A summary of these different approaches is given in (4). This paper is limited to the examination of analytic models.

3. *The Linear Decision Rule*

3.1. *The HMMS-Model*

The Linear Decision Rule (LDR) was developed by Holt, Modigliani, Muth and Simon as a means of making *aggregate employment* and *production* rate decisions, and the model was first tested in a paint factory (12). The LDR is based on the development of a quadratic cost function with cost components made up to regular payroll, hiring, layoff, overtime, inventory holding, back ordering and machine setup costs. The quadratic cost function if then used to derive two linear decision rules for setting work force levels and production rate for the upcoming period based on a forecast of aggregate sales for 12 periods (months) ahead. We will summarize the structure of the model.

Let be

T = planning horizon of T periods ($t = 1 \ldots T$)
S_t = demand forecast, period t
W_t = size of workforce, period t
I_t = inventory on hand-backorders, end of period t
P_t = production level, period t

An optimal (P_t, W_t) policy has to be determined. It is clear that fluctuations in demand can be met by continuously adjusting production ($P_t = S_t$). Such a policy clearly results in important workforce adjustments which are not desirable per se (hiring + firing cost, poor worker morale, low efficiency). On the other hand, maintaining a constant workforce results in considerable inventory costs.

The structure of the cost function, period t, as tested by HMMS is given by following terms :

- regular payroll costs = $C_1 W_t$ (3.1)
- hiring + firing costs = $C_2 (W_t - W_{t-1})^2$ (3.2)
- overtime costs = $C_3 (P_t - C_4 W_t)^2 + C_5 P_t - C_6 W_t$ (3.3)

- inventory costs = $C_7\,(I_t - C_8 - C_9 S_t)^2$ (3.4)

We then obtain the following problem formulation :

$$\text{Minimize } \sum_{t=1}^{T} C_t \tag{3.5}$$

subject to the inventory constraints

$$I_{t-1} + P_t - S_t = I_t \quad \text{for } t = 1 \ldots T \tag{3.6}$$

$$\text{and } P_t \geq 0,\ W_t \geq 0 \tag{3.7}$$

where

$$C_t = C_1 W_t + C_2 (W_t - W_{t-1})^2 + C_3 (P_t - C_4 W_t)^2$$

$$+ C_5 P_5 - C_6 W_6 + C_7 (I_t - C_8 - C_9 S_t)^2 \tag{3.8}$$

The constrained optimization problem (3.5) - (3.7) may be solved by the Lagrangian technique, which yields the following objective function

$$G = \sum_{t=1}^{T} C_t + \sum_{t=1}^{T} \lambda_t \,(I_{t-1} - I_t + P_t - S_t) \tag{3.9}$$

where λ_t are Lagrangian multipliers.

Differentiation of (3.9) partially with respect to the unknown variables yields a system of LDR's of following type

$$P_t = \sum_{t}^{t+T} \alpha_t S_t + k_1 W_{t-1} + k_2 - k_3\, I_{t-1} \tag{3.10}$$

$$W_t = \sum_{t}^{t+T} \beta_t S_t + k_4 W_{t-1} + k_5 - k_6\, I_{t-1} \tag{3.11}$$

where the α's, β's and k's are coefficients which take on particular values for specific industrial situations, remaining fairly stable as long as reasonably similar circumstances prevail. Equations (3.10) and (3.11) would be used at the beginning of each period (month). Both equations are extremely easy to compute. They involve a weighted forecast of sales as well as beginning workforce and inventory levels (start of period t = end of period t-1).

3.2. *Comments*

The HMMS-model has achieved considerable prominence in the literature. Nevertheless it has not been extensively adopted in industry for different reasons :

i) the difficulty of estimating cost factors (especially with quadratic assumptions which do not always apply)

ii) the homogeneous workforce

iii) the need to express all jobs in aggregate figures, using standard processing hours

iv) the hypothesis that production can be started and completed in the same period.

Clearly, different results will be obtained depending upon how we set the planning horizon T, and how often we compute the coefficiants α_t and β_t. It is obvious that the ability to predict deteriorates when T is too long. Therefore, proper validation of the model is needed.

3.3. *Extensions of the LDR*

A) The following important property was given by Simon (19) : *If demand is stochastic, and if the objective is to minimize expected cost over the planning horizon, and if the costs are, in fact, quadratic, then the LDR is optimal.*

B) Sypkens (20) developed an extension of the LDR which includes plant capacity as a decision variable in addition to W and P. There are some instances where fundamental physical capacity adjustments can also be made (cfr. 2 parallel production units, each able to perform the operations). A model similar to the HMMS-model was developed and three LDR's were obtained (with respect to P, W and C).

C) Chang and Jones (6) generalized the LDR methodology in a multiproduct environment where production cannot be started and completed in the same period. They introduce a labor distribution matrix D_{jt}, which specifies the fraction of total labor effect required for product j in period t. The model provides for following costs : hiring, lay-off, payroll, idle time, overtime and inventory. The costs are also assumed to be quadratic thereby permitting the model to be solved using differential calculus combined with a library computer program which solves simultaneous linear equations. A disaggregated print-out (per product) is obtained.

4. *The Linear Programming Model*

4.1. *The Model*

Different LP-approaches have been presented in the literature in order to solve the medium term planning problem (11). The main advantages of such models over the HMMS model are :

(a) they don't require the assumption of quadratic costs

(b) they are much simplier to construct and to manipulate

(c) we have the powerful LP-theory to our disposal

(d) disaggregation of the variables is relatively easy.

Let following symbols be defined :

T = planning horizon

S_t = demand forecast, period t

W_t = size of workforce, period t

δ_t^+ = increase in workforce, period t

δ_t^- = decrease in workforce, period t

c^+ = hiring cost

c^- = firing cost

I_t = inventory on hand-backorders, at the end of period t

X_t = actual hours of regular time production, period t

A_t = regular capacity available per employee, period t

a = cost of regular time production per hour

Y_t = actual hours of overtime production, period t

B_t = overtime capacity available per employee, period t

b = cost of overtime production per hour

h = inventory holding cost.

The objective function to be minimized is total cost. Here such cost is treated as being reasonably well represented by a linear function consisting of some combination of payroll, hiring, lay-off, overtime and inventory costs. Constraints are set on the availability of regular and overtime production capacity, the amount of capacity that can be added or removed, and restrictions on inventories that are in line with meeting demand. Standard units are required to achieve comparability of inputs and outputs in aggregation. The standard unit will be given in terms of manhours required. A description of the constraints and objective function is given below :

Regular production constraint

$$X_t \leq A_t \cdot W_t \quad \text{for all } t = 1 \ldots T \qquad (4.1)$$

Overtime production constraint

$$Y_t \leq B_t \cdot W_t \quad \text{for all } t = 1 \ldots T \qquad (4.2)$$

Inventory constraint

Let I_o be the starting inventory level.
Then it follows

$$I_1 = I_o + X_1 + Y_1 - S_1 \text{ , and}$$

$$I_2 = I_1 + X_2 + Y_2 - S_1 = I_o + (X_1 + X_2) + (Y_1 + Y_2) - (S_1 + S_2)$$

$$\vdots$$

$$I_t = I_o + \sum_1^t X_t + \sum_1^t Y_t - \sum_1^t S_t \text{ for all } t \tag{4.3}$$

Capacity change constraints

$$W_t - W_{t-1} = \delta_t^+ - \delta_t^- \quad \text{for all } t = 1 \ldots T \tag{4.4}$$

Recall that any number (+ or -) can be represented as the difference between two *non-negative* numbers (δ_t^+ and δ_t^-). In the general case ($W_t - W_{t-1}$) can be represented by an infinite number of different values of δ_t^+ and δ_t^- by simply adding the same constant to both. It is clear however that, with respect to our objective function, the optimum program will not contain both variables at a positive level.

Non-negativity constraints

$$X_t, Y_t, \delta_t^+, \delta_t^-, I_t, W_t \geq 0 \quad \text{for all } t = 1 \ldots T \tag{4.5}$$

We now formulate the objective function. Let C_t be the cost in period t.
We then obtain :

$$C_t = c^+ \delta_t^+ + c^- \delta_t^- + a.X_t + b.Y_t + h.I_t \tag{4.6}$$

where δ_t^+ , δ_t^- , X_t , Y_t , I_t and W_t are unknown variables to be determined according to following objective

$$\text{Minimize } C = \sum_{t=1}^{T} C_t \tag{4.7}$$

Equations (4.1) - (4.7) determine a straightforward LP for which algorithms are available. Since the number of employees is integral, we can restrict the solution to integer values of W_t, δ_t^+ and δ_t^- . The resultant would be a mixed integer linear program (MILP). Generally, this restriction is unnecessary because the lack of accuracy of the data we are using renders such a refinement meaningless. Moreover, computerprograms for solving large MILP's are less well developed then for solving simple LP's. Generally one uses appropriate rounding techniques after having obtained the LP solution.

Some simplification of the above model is possible by simply substituting equation (4.3) in (4.6) and (4.7).
We then obtain :

$$\text{Min } C = \sum_{t=1}^{T} \{c^+ \delta_t^+ + c^- \delta_t^- + a.X_t + b.Y_t$$

$$+ h [\sum_1^t (X_t + Y_t - S_t) + I_o]\} \qquad (4.8)$$

subject to

$$X_t \leq A_t . W_t \qquad (4.1)$$

$$Y_t \leq B_t . W_t \qquad (4.2)$$

$$W_t - W_{t-1} = \delta_t^+ - \delta_t^- \qquad (4.4)$$

$$I_o + \sum_1^t (X_t + Y_t - S_t) \geq 0 \qquad (4.9)$$

$$X_t, Y_t, \delta_t^+, \delta_t^- \text{ and } W_t \geq 0 \qquad (4.10)$$

The above constraints apply to all $t = 1 \ldots T$. The size of the resulting LP is rather modest : we have 4T constraints in 5T original variables.

4.2. Extensions

The above model can be enriched in several ways to take into account other managerial restrictions or to incorporate other cost factors. We limit ourselves to a few such examples :

A) Let e be the cost of idle time (per hour). Writing equation (4.1) in standard form yields

$$X_t - A_t . W_t + S_{1t} = 0 \qquad (4.11)$$

where the slackvariable S_{1t} represents the amount of idle time. The objective function has to be completed with a term $e.\Sigma S_{1t}$.

B) Any restrictions on the number of employees hired or laid off can easily be incorporated :

$$\delta_t^+ \leq \alpha_t W_{t-1} \quad \text{for } t = 1 \ldots T \qquad (4.12)$$

$$\delta_t^- \leq \beta_t W_{t-1} \quad \text{for } t = 1 \ldots T \qquad (4.13)$$

where α_t and β_t are given.
It is clear that even the timing of hiring and firing can be influenced by constraints of the above type.

C) A limitation of overtime - as a function of regular time e.g. - can easily be included :

$$Y_t \leq \gamma_t \cdot X_t \quad \text{for } t = 1 \ldots T \tag{4.14}$$

where γ_t is given.

D) Different types of overtime can be included. Suppose e.g. that per unit overtime cost is b_1 for $0 \leq Y_t \leq B_t^1 \cdot W_t$ whereas overtime cost becomes b_2 above the $B_t^1 \cdot W_t$ limit.
In this cas the variable Y_t is simply disaggregated through following procedure :

$$Y_t = Y_t^1 + Y_t^2 \tag{4.15}$$

where $0 \leq Y_t^1 \leq B_t^1 \cdot W_t$

and where $(b_1 \cdot Y_t^1 + b_2\, Y_t^2)$ replaces $b\, Y_t$ in the objective function.

Note that this transformation yields the possibility of introducing the fact that "a small change does not cost as much as a large one" (cfr. HMMS-Model !)

4.3. *The Disaggregated Model*

For simplicity of exposition, the above LP model was presented as to optimize *aggregate* production, inventory and workforce. It is obvious however that the LP model offers a high degree of freedom with respect to disaggregation. In particular, we can optimize the objective function, taking into account the individual requirements of N products or groups of products.

Let us redefine the variables X_{jt}, $Y_{j,t}$, $\delta^+_{j,t}$, $\delta^-_{j,t}$ and $W_{j,t}$ to represent their respective quantities relative to product $j = 1 \ldots N$. The problem then becomes :

$$\text{Minimize } C = \sum_{j=1}^{N} \sum_{t=1}^{T} C_{j,t} \tag{4.16}$$

where

$$c_{j,t} = c_j^+\, \delta^+_{j,t} + c_j^-\, \delta^-_{j,t} + a_j \cdot X_{j,t} + b_j \cdot Y_{j,t}$$
$$+ h_j \left[\sum_1^t (X_{j,t} + Y_{j,t} - S_{j,t}) + I_{j,o} \right] \tag{4.17}$$

subject to

$$X_{j,t} \leq A_{j,t} \cdot W_{j,t} \tag{4.18}$$

$$Y_{j,t} \leq B_{j,t} \cdot W_{j,t} \tag{4.19}$$

$$W_{j,t} - W_{j,t-1} = \delta^{+}_{j,t} - \delta^{-}_{j,t} \qquad (4.20)$$

$$I_{j,o} + \sum_{1}^{t} (X_{j,t} + Y_{j,t} - S_{j,t}) \geq 0 \qquad (4.21)$$

$$X_{j,t} \ / \ Y_{j,t} \ / \ \delta^{+}_{j,t} \ / \ \delta^{-}_{j,t} \ / \ W_{j,t} \geq 0 \qquad (4.22)$$

Constraints (4.18) - (4.22) apply to all periods t = 1 ... T and to all products j = 1 ... N

The size of the program is necessarily expanded to 4NT constraints in 5NT original variables.

A further disaggregation of the LP model (over labor classes and machine groups) was used by Shwimer (17).

4.4. Comments

The LP-formulation given above is clearly an extremely powerful model for smoothing of production, workforce and inventory. The powerful techniques and results of linear programming are made available to the analyst and the manager. Standard computer packages are available. As a result, one might question the utility of the HMMS-model.

5. *The Distribution Model*

5.1. The Model

Bowman (3) proposed a "distribution" or "transportation" formulation of the aggregate planning problem. Let therefore be

- T = decision horizon
- S_t = sales forecast, period t
- I_t = stocklevel at end of period t
- R_t = regular time production capacity, period t
- O_t = overtime production capacity, period t.

The sales demand during each time period may be met from one of the following sources : *stock, regular production* or *overtime production.* This idea yields a transportation formulation, where goods available in certain sources, are to be shipped to certain destinations.

The associated costs are :

- C_R = per unit production cost on regular time
- C_O = per unit production cost on overtime
- C_I = per unit storage cost (per period).

We might further assume that items produced during period t are not available to the customer until period (t+1). Consequently, some initial stock must be available to satisfy demand for salesperiod 1. We can now represent the transportation matrix as follows (for the sake of easy illustration we took T = 4) :

Production Period		Sales Period 1	2	3	4	Final stock	Total capacity
0	Opening stock	0	C_1	$2C_1$	$3C_1$	$4C_1$	I_o
1	Regular prod.	-	C_R	C_R+C_1	C_R+2C_1	C_R+3C_1	R_1
	Overtime prod.	-	C_o	C_o+C_1	C_o+2C_1	C_o+3C_1	O_1
2	Regular prod.	-	-	C_R	C_R+C_1	C_R+2C_1	R_2
	Overtime prod.	-	-	C_o	C_o+C_1	C_o+2C_1	O_2
3	Regular prod.	-	-	-	C_R	C_R+C_1	R_3
	Overtime prod.	-	-	-	C_o	C_o+C_1	O_3
Total demand during period		S_1	S_2	S_3	S_4		

Obviously, it is not necessary that the costs C_R, C_o and C_1 should be the same for each period.

5.2. *The Multi-Product Model*

The distribution method can easily be extended to cover production planning for two or more products merely by constructing two or more columns for each of the sales periods and for the final stock. A condition however is that common units can be established for all products to express sales, and production and storage costs.

5.3. *Comments*

The distribution method does not account for *capacity change* costs. Also, the program results are not all in the most desired form, for one must figure the number of workers to be hired or laid-off. Moreover all costs are assumed to be *linear*.

Major advantages of the distribution method are :

- the availability of good *computer packages*
- the *flexibility* of the model with respect to extensions (different products, subcontracting, cost modifications, etc ...)
- the program results relate directly to the obvious factors of the problem.

6. *Conclusion and Future Trends*

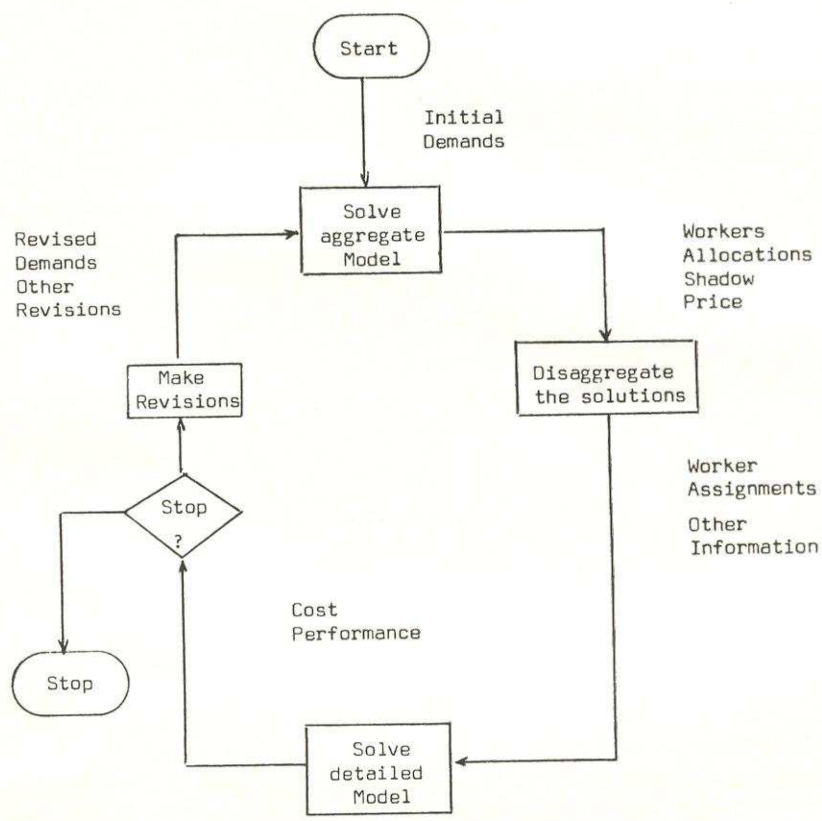
Analytical methods of production planning form the basis for continued development of newer methods. These models will continue to provide a standard for comparing the effectiveness of new approaches to the problem, simply because these methods provide optimum solutions to specific test situations. The difficulties with analytical methods are in the requirements that cost and/or revenue functions must be expressed as either quadratic or linear relationships, thus limiting the realism which can be incorporated in the model. This is the reason why heuristic methods (cfr. (2), (13) and (22)) will continue to be extensively examined. Says Buffa (4) : "*We face the tradeoff between the desirability of obtaining a known optimum solution to a relatively simplified model versus obtaining a near optimum solution to a richer, more realistic model.*"

It should be mentioned here that very recently succesful results have been obtained by different people in trying to coordinate medium term and short term planning objectives, e.g. :

- Newson (15) has dealt with a linear programming aggregate model and a Wagner-Within (23) type detail model in looking at the capacitated lotsize problem. He found that iteration between these two models could improve the overall results.

- Green (10) has dealt with a HMMS-type aggregate model and a relatively simple job-shop simulation at the detailed level. The detail model is guided by the aggregate model and used to evaluate conditions for this model. The coupling of both models is an attempt to enable the aggregate model to use exogeneous information. Green found that feedback from the detail model, in particular regarding a specific productivity parameter in the aggregate objective function, was helpfull in improving the overall shop performance.

- Shwimmer (17) has further corroborated the benefits to be gained by coordination of aggregate and detail scheduling decisions in the job shop context. He establishes an aggregate model which is a mixed integer linear program. However the linear approximation is sufficient in practice (when appropriate rounding techniques are used). The complexity of his detail model precludes the use of an optimum seeking method. Therefore, heuristically based job dispatching (e.g. COVERT) and labor assignment rules within a simulation framework are used to obtain a good, but not necessarily optimum solution. Iteration between these two submodels leads to a good overall solution :



- Gelders and Kleindorfer ((8) and (9)) established an optimum seeking algorithm for coordinating aggregate and detailed scheduling in a one-machine job-shop. The fundamental thrust of their work was that the results obtained in relatively simple cases (for which optimum seeking methods can be elaborated) very often provide insight into the basic structural aspects of the problem. A branch and bound algorithm was designed. This algorithm examines explicitely or implicitely all possible combinations of capacity decisions on the aggregate level *and* sequence decisions on the detail level. The lower bounding proding problem for this BB-algorithm is a relatively simple ILP.

The implication is that, although the optimum seeking method may be unfeasible for large scale problems, real benefit may be gained from the possibility of delineating an excellent capacity plan $\underline{x}$ at low computational cost.

As a matter of fact, the complexity of the detailed scheduling problem led to the generalized use of dispatching rules (applied to the capacity plan derived from the medium term model). A considerable improvement may be expected when applying these dispatching rules at capacity plan $\underline{x}$ instead of the capacity plan resulting from the aggregate model alone.

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