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# Returns interval and variability in risk measurement

A. Corhay

Department of Management, FEGSS
University of Liège
7, Bld du Rectorat, B31
4000 Liège, Belgium

A. Tourani Rad

Limburg Institute of Financial Economics
Maastricht University
P.O. Box 616
6200 Maastricht, The Netherlands

**Abstract**

In this paper we show the variability of beta estimates for each differencing interval. Betas depend on the manner daily prices are juxtaposed to calculate runs. A method is proposed to reduce this variability.

**Keywords** : systematic risk, intervalling effect, return interval

## RETURNS INTERVAL AND VARIABILITY IN RISK MEASUREMENT

### 1 Introduction

The sensitivity of the estimate of the systematic risk of a security to the length of the differencing interval used to measure the returns has received considerable attention in the finance literature. The impact of the length of the differencing interval used to measure the returns on the estimated betas was first shown by Pogue and Solnik in 1974. Using samples from seven European countries, they found that the daily beta estimates depend on the length of the differencing interval. The first models dealing with the intervalling effect bias in beta were presented by Scholes and Williams (1977) and Dimson (1979). Both imputed the intervalling effect bias in beta to infrequent trading. Beyond the difficulty of obtaining highly synchronous data, these models do not take account of delays in the price adjustment which can be caused by the microstructure of the market and the reaction of investors to the arrival of information. This aspect has been incorporated in the model of Cohen, Hawawini, Maier, Schwartz and Whitcomb, henceforth CHMSW (1980). According to their theory the expected magnitude of the price-adjustment delays is related to the thinness of the securities: thinner securities have greater adjustment delays than frequently and highly traded securities. They also demonstrated that thin securities have a downward bias in their betas for short differencing intervals, while relatively frequently traded securities have an upward bias.

While these studies focuse on the impact and adjustment of the differencing interval length on the systematic risk estimate, none has addressed the variability of these estimates caused by the juxtaposition of prices to calculate returns. This is the objective of this paper. We investigate the behaviour of betas of 50 Dutch firms as a function of the return measurement interval. We show that there is an intervalling effect and also that there exists variability in beta coefficients due to the juxtaposition of prices for any specified interval length. A correction method is then proposed in order to reduce this variability.

## 2 Data

The sample we use in this study consists of 50 firms randomly chosen excluding the five biggest firms since they represent more than two third of the market capitalization of the Dutch firms listed on the Amsterdam Stock Exchange. The daily closing prices for a period starting 28/10/87 and ending 1/3/92 are used to calculate the returns. These prices are adjusted for dividends and capital modifications. Returns for any interval length are calculated as the difference between the natural logarithm of two closing prices, $R_t = \ln(P_t + D_t) - \ln(P_{t-1})$. The CBS general index, the most appropriate index available in the Netherlands, is used as a proxy for the market index.

## 3 Interval Length and Systematic Risk Estimates

We assume that the security returns are generated by the Market Model, that is,

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it} \quad t=1,....,T \tag{1}$$

where $\beta_i$, the security beta, measures the change in $R_{it}$ as a result of a change in the market index return $R_{mt}$, $\alpha_i$ measures the change in $R_{it}$ that is independent of a change in $R_{mt}$, and $\varepsilon_{it}$ is the random error term. According to this model, neither $\alpha_i$ nor $\beta_i$ should depend on the length of the differencing interval used to calculate the returns. However, Hawawini (1980) established the relationship between the beta of security i, and the length of the differencing interval, L, when returns are defined as continuously compounded returns. It is:

$$\beta_i(L) = \beta_i(1) \frac{L + \sum_{s=1}^{L-1} (L-s) \dfrac{\rho_{im}^{+s} + \rho_{im}^{-s}}{\rho_{im}^{0}}}{L + 2 \sum_{s=1}^{L-1} (L-s)\, \rho_m^s} \tag{2}$$

where $\beta_i(1)$ is the systematic risk estimated using one day intervals, $\rho_{im}^0$, $\rho_{im}^{+s}$, $\rho_{im}^{-s}$ are respectively the intertemporal cross-correlation coefficient of order 0, +s (lead) and -s (lag) between the returns, measured on a one-day differencing interval, of security i and the market, and $\rho_m^s$ is the autocorrelation of order s on the market daily returns. As the intertemporal cross-correlation and the market autocorrelation generally decrease with the order of the lag, the value of the OLS security beta approaches an asymptotic value when the differencing interval is lengthened.

$$\beta_i = \lim_{\substack{L\to\infty \\ T\to\infty}} \beta_i(L) \qquad (3)$$

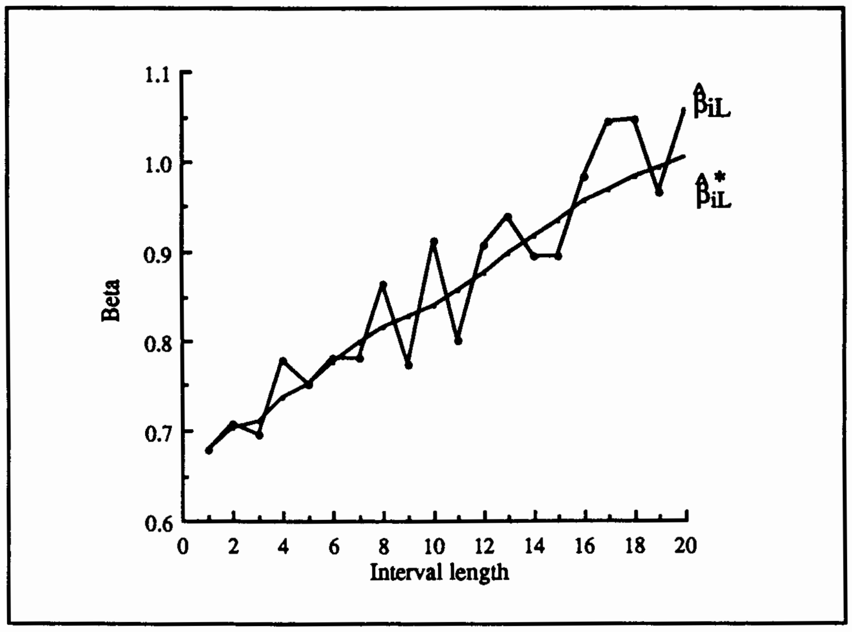
The estimates of the systematic risk for each security and for differencing interval lengths varying from 1 to 20 days have been calculated using the Market Model.

$$R_{iLt} = \hat{\alpha}_{iL} + \hat{\beta}_{iL} R_{mLt} + \varepsilon_{iLt} \qquad \text{for } L=1,\dots,20 \text{ and } t=1,\dots,T. \qquad (4)$$

where $R_{iLt}$ and $R_{mLt}$ are respectively the returns of security i and the market index, measured over a differencing interval of L days, L varying from one day to twenty days.

The average beta for the sample is plotted in Figure 1. As expected it goes up from .6767 for a one day interval to 1.0550 for a twenty days interval.

Figure 1 : Average Beta Coefficients




An interesting feature of (2) is that it shows that the value of the systematic risk for any specified interval length directly depends on the index specification, or more precisely, on both the intertemporal cross-correlations between the index and the security returns and on the autocorrelation in the market index returns, that is, indirectly, on the delay in the price adjustment. Since the autocorrelation in the market index returns can depend on which day of the week the period used to measure the returns starts, one could also expect that the systematic risk is also dependent on the day of the week. That is, beta coefficients depends on the manner daily prices are juxtaposed to calculate returns on intervals longer than one day. Since a return for a specific interval length is measured as the difference in logarithm between two well-defined daily prices, any price move, whatever its magnitude, that occurs and is wiped out between these two days does not enter into the calculation of the return, nor, consequently, into the estimation of the beta. On the other hand, substantial moves that systematically occur on the day returns are measured have an impact on estimated betas. As an example, the weekly beta coefficients, i.e. beta coefficients when returns are measured on a weekly basis have been estimated. It is clear that their values, that are reported in table 1, depend on the day of the week used to calculate the returns.

Such variations in the beta coefficients indicates that a correction of the $\beta_i(L)$ is necessary to better discern the convergence of the coefficients. It consists in running the regression L times for an interval length of L and in calculating an average beta coefficient. Such procedure allows us to avoid too high and too low estimated beta coefficients which would be due only to the juxtaposition of the daily prices. The regression is run a first time with returns of interval length L calculated using the complete series of daily returns. Then the first daily return is deleted, the returns of interval length L are recalculated with the remaining observations and the regression is run again and so forth until it is run L times. So the regression model becomes:

$$R_{iLnt} = \hat{\alpha}_{iLn} + \hat{\beta}_{iLn} R_{mLnt} + \varepsilon_{iLnt} \quad \text{for } n=1,\ldots,L. \tag{5}$$

For each interval length, the average beta $\hat{\beta}^*_{iL}$ is then calculated:

$$\hat{\beta}^*_{iL} = \sum_{n=1}^{L} \hat{\beta}_{iLn} \Big/ L \tag{6}$$

The average estimate of systematic risk based on equation (6) for all twenty intervals is plotted in Figure 1. Although the behaviour of the coefficients has the same pattern as before, that is, it increases with the interval length, the variability decreases and it is much smooth. Therefore, using this method allows to avoid extreme values of the betas which could be due only to the juxtaposition of the prices.

In order to examine the speed of convergence of the betas estimated using equations (1) and (6), we compare them to their asymptotic values obtained by a second pass regression as in Cohen et al. (1983) study.

$$\hat{\beta}_{iL} = \hat{\beta}_i + \hat{\gamma}_i f(L^{-n}) + \upsilon_{iL} \tag{7a}$$

$$\hat{\beta}^*_{iL} = \hat{\beta}^*_i + \hat{\gamma}^*_i f(L^{-n}) + \upsilon^*_{iL} \tag{7b}$$

$\hat{\beta}_i$ and $\hat{\beta}^*_i$ are estimates of the asymptotic beta coefficient and $\hat{\gamma}_i$ and $\hat{\gamma}^*_i$ are measures of the intervalling effect on the beta of security i.[1] Estimated asymptotic betas are 0.9548 and 0.9376, and intervalling effect coefficients are -0.3915 and -0.3678 based on equation (7a) and (7b) respectively.

Table 2 : Mean Squared Errors

| Interval length | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\hat{\beta}_{iL}$ | 8.6309 | 7.8220 | 6.3073 | 5.1313 | 4.6125 | 3.9964 | 2.7998 | 2.0129 | 2.9696 | 1.5456 |
| $\hat{\beta}^*_{iL}$ | 7.8170 | 6.2224 | 5.3897 | 4.2436 | 3.3287 | 2.5397 | 1.9585 | 1.4582 | 1.0765 | 0.7989 |
| Interval length | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| $\hat{\beta}_{iL}$ | 2.4306 | 1.2500 | 1.9422 | 1.0781 | 1.5537 | 1.1699 | 1.7346 | 1.2020 | 1.8444 | 1.6816 |
| $\hat{\beta}^*_{iL}$ | 0.5851 | 0.3560 | 0.16671 | 0.0604 | 0.0368 | 0.0660 | 0.1229 | 0.2098 | 0.3046 | 0.4555 |

The appropriateness of equation (7b) is further confirmed by the value of the mean square errors (MSE) of its estimated betas from its respective asymptotic one. These are always lower than their OLS equivalent usually used in the literature. These MSE are reported in table 2. One can observe that asymptotic beta coefficients are quite close using both methods, but the average

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1 Various inverse functions of L were tested for the Dutch market, that is, $L^{-n}$, $\ln(1+L^{-n})$, $\ln(1+L)^{-n}$ and $\exp(-L^{-n})$, and it appeared that the function $L^{-.8}$ fits the data best.

method provides better beta estimates whatever the length of the differencing interval than the alternative method used in finance literature.

## 4 Conclusion

In this paper we investigated the behaviour of betas of Dutch firms as a function of the return measurement interval. We found beta estimates measured from different intervals differ significantly from each other and those from short intervals are usually lower than those obtained with longer intervals. Moreover, it is shown betas depend on the manner daily prices are juxtaposed to calculate the returns. A way to account for this variability is to average the different betas for each interval length.

## 5 References


Cohen, K.J., Hawawini, G.A., Maier, S.F., Schwartz, R.A. and D.K. Whitcomb, 1980, Implications of Microstructure Theory for Empirical research on stock Price Behavior, Journal of Finance 35, 249-257.

Cohen, K.J., Hawawini, G.A., Maier, S.F., Schwartz, R.A. and D.K. Whitcomb, 1983, Estimating and Adjusting for the Intervalling Effect Bias in Beta, Management Science 29, 135-148.

Corhay, A. (1992), 'The Intervalling Effect Bias in Beta: a Note', *Journal of Banking and Finance* 16, 61-73.

Dimson, E., 1979, Risk Measurement when Shares are Subject to Infrequent Trading, Journal of Financial Economics 7, 197-226.

Fowler, D.J. and C.H. Rorke, 1983, Risk Measurement when Shares are Subject to Infrequent Trading: Comment, Journal of Financial Economics 12, 279-283.

Hawawini, G.A., 1980, Intertemporal Cross-Dependence in Securities Daily Returns and the Short-Run Intervalling Effect on Systematic Risk, Journal of Financial and Quantitative Analysis 15, 139-149.

Pogue, G.A. and B.H. Solnik (1974), 'The Market Model Applied to European Common Stocks: Some Empirical Results', *Journal of Financial and Quantitative Analysis* 9, 917-944.

Scholes, M. and J. Williams, 1977, Estimating Betas from Nonsynchronous Data, Journal of Financial Economics 5, 309-327.

**Table1 : Weekly Beta Coefficients**

| Firm | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| 1 | 0.6120 | 0.3507 | 0.4956 | 0.5271 | 0.6693 |
| 2 | 1.0822 | 1.1300 | 1.2439 | 0.9636 | 1.1752 |
| 3 | 1.1024 | 1.2424 | 1.2693 | 1.1273 | 1.0521 |
| 4 | 1.1633 | 1.1184 | 0.9160 | 1.0591 | 1.1903 |
| 5 | 0.7907 | 1.1941 | 1.2670 | 0.9395 | 0.9517 |
| 6 | 1.4276 | 1.4442 | 1.2678 | 1.2423 | 1.2713 |
| 7 | 0.8025 | 0.8359 | 0.7266 | 0.8968 | 0.7962 |
| 8 | 0.5928 | 0.6679 | 0.6082 | 0.5663 | 0.6724 |
| 9 | 0.8975 | 1.3593 | 1.5012 | 0.6857 | 0.8355 |
| 10 | 0.2819 | 0.0568 | 0.3390 | 0.2551 | 0.2751 |
| 11 | 0.6227 | 0.3484 | 0.5260 | 0.5000 | 0.4701 |
| 12 | 1.2418 | 1.3890 | 1.3758 | 1.2835 | 1.4685 |
| 13 | 0.4299 | 0.5079 | 0.3953 | 0.2051 | 0.5251 |
| 14 | 0.9419 | 0.8977 | 1.0826 | 1.0694 | 1.1046 |
| 15 | 0.3451 | 0.6803 | 0.5667 | 0.7100 | 0.5852 |
| 16 | 0.5543 | 0.5499 | 0.6341 | 0.7200 | 0.8590 |
| 17 | 1.2890 | 1.4383 | 1.4885 | 1.4969 | 1.3303 |
| 18 | 0.3386 | 0.3531 | 0.4097 | 0.3979 | 0.4005 |
| 19 | 1.0641 | 1.0779 | 1.0286 | 0.8591 | 0.9775 |
| 20 | 1.2343 | 0.9712 | 1.5692 | 1.6225 | 1.3350 |
| 21 | 1.2366 | 1.2378 | 1.2354 | 1.3801 | 1.4893 |
| 22 | 0.8315 | 0.8818 | 0.7049 | 0.7931 | 0.7622 |
| 23 | 0.5801 | 0.9026 | 0.5178 | 0.4541 | 0.6034 |
| 24 | 0.4146 | 0.6519 | 0.6495 | 0.9606 | 0.6534 |
| 25 | 0.2402 | 0.2604 | 0.4295 | 0.3169 | 0.3758 |
| 26 | 0.7987 | 0.8346 | 0.7424 | 0.6121 | 0.6184 |
| 27 | 0.6445 | 0.5797 | 0.4810 | 0.5205 | 0.7021 |
| 28 | 1.0531 | 1.0291 | 0.7014 | 0.7057 | 0.7601 |
| 29 | 0.4856 | 0.9997 | 0.9054 | 0.4399 | 0.4865 |
| 30 | 0.7998 | 0.7324 | 0.8118 | 0.7301 | 0.6378 |
| 31 | 0.5082 | 0.1527 | 0.2719 | 0.4519 | 0.3102 |
| 32 | 1.3535 | 1.3825 | 1.3836 | 1.2079 | 1.3049 |
| 33 | 0.1541 | 0.2675 | 0.1594 | 0.1362 | 0.1244 |
| 34 | 0.0352 | 0.0329 | 0.0306 | -0.0077 | -0.0302 |
| 35 | 0.4680 | 0.7182 | 0.9158 | 0.7777 | 0.5929 |
| 36 | 0.6998 | 0.7685 | 0.8211 | 0.8290 | 0.8023 |
| 37 | 0.0945 | 0.1919 | 0.2761 | 0.2091 | 0.1247 |
| 38 | 0.4006 | 0.4443 | 0.6280 | 0.7306 | 0.4847 |
| 39 | 0.7174 | 0.7726 | 0.8794 | 0.4433 | 0.7751 |
| 40 | 0.0035 | -0.0466 | 0.0314 | 0.0624 | -0.0089 |
| 41 | 0.3691 | 0.8618 | 1.0731 | 0.8145 | 0.9228 |
| 42 | 0.7244 | 0.6896 | 0.8062 | 0.7814 | 0.8497 |
| 43 | 0.3931 | 0.2450 | 0.3861 | 0.2749 | 0.3796 |
| 44 | 0.8437 | 0.7969 | 1.1443 | 0.7451 | 0.9062 |
| 45 | 1.3677 | 1.4651 | 1.4062 | 0.9579 | 1.0025 |
| 46 | 0.3880 | 0.2560 | 0.4841 | 0.5407 | 0.5394 |
| 47 | 1.0258 | 1.1032 | 0.9360 | 1.1088 | 1.0795 |
| 48 | 1.1038 | 1.1882 | 1.2773 | 0.9819 | 0.9181 |
| 49 | 1.1056 | 1.0131 | 0.8823 | 1.2208 | 1.0779 |
| 50 | 0.5849 | 0.7532 | 0.7512 | 0.5583 | 0.5729 |