

Traffic crowding in urban areas  
Control with urban tolls and flexible working hours

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**Abstract**

*The paper analyses the effect of a toll and flexible working hours on the morning traffic congestion caused by car commuters in an urban area. According to the economic approach the mechanism of congestion results from the attitudes of individual commuters making an arbitrage between schedule and travel costs. The congestion is the collective externality generated by this individual behaviour. The approach proposed in the paper consists in modelling the commuters' preferences for a particular departure time. Four incentives are explicitly taken into account. The two schedule incentives are based on the departure time from home and the arrival time in the office. The two travel incentives are based on the congestion time and the peak-hour toll. It is shown that the confrontation between the incentives leads to equilibrium distributions of both schedule and travel times. The benefits of peak-hour toll and working time flexibility for reducing the morning congestion are investigated.*

**Keywords:** morning car congestion, peak-hour toll, flexible working time, preference modelling

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## 1. Introduction

A major contemporaneous problem in all industrialised countries is the congestion in urban areas. Especially the traffic jams in the morning and the evening when car commuters moves from and to their domiciles in the green outskirts to the Central Business District (CBD) have reached an unacceptable amplitude.

The objective of the paper is to analyse the effect of an urban toll and flexible working hours on the morning traffic congestion caused by private car commuters. The starting point will be the economic congestion model introduced by K. Small (1982, 1983, 1992). This model developed in section 2 considers that morning congestion is a social cost, i.e. a social externality of the car driving. It results from the egoistic individual behaviour of the commuters to the CBD. The internalisation of this social cost provides the time-dependent value on the toll to be charged on car commuters during the peak-hours. Note that toll policies are today operational in several cities in Asia (Wilson (1988)) and are being seriously considered in continental Europe as well.

A simple case is analysed by considering commuting times between 6 a.m. and 12 a.m. and a single lane with limited capacity between a unique green suburb and the CBD.

This basic economic model is not very transparent with respect to the incentives responsible for the commuters' behaviour, i.e. their choice of a particular departure time from home. Therefore it is extended in section 3 by elaborating multiple time-dependent incentives for commuters to choose a particular time for leaving their home within the considered time window. Four different strategic scenarios are considered with or without toll, combined with or without flexible working time. In each case these time-distributions are aggregated into one representing on a statistical basis the departure distribution from home. It is shown that in some cases a final equilibrium distribution results from the day-to-day experience of commuters with the road congestion conditions.

A provisional conclusion and some indication of further research work on improved commuters' modelling is provided in section 4.

## 2. The economic approach with the internalisation of social costs

A simple traffic congestion model has been described by K. Small in "Urban transportation economics" (1992). It is further elaborated in Mirabel (1999). In the following this approach will be labelled "*economic model*".

The setting is the use of a unique road lane by car driving commuters living in a unique green suburb away from the polluted CBD. In this idealised city, the car commuters get immediately from their houses on the lane and from there to their offices downtown. In a more advanced modelling the bottleneck effects rising up at the entrance and exit of the highway could easily be incorporated into the model. The existence of a unique lane is a large simplification though it seems to be sufficient as a first approximation of the congestion mechanism. This assumption could easily be relieved.

The result is a massive crowding in the morning (and symmetrically in the evening) on the connecting lane to (and from) the CBD. Small's model examines the behaviour of commuters with respect to their home departure times to office during the morning rush hours only. There are two sub-models:

- The *individual sub-model* examines the commuter behaviour in the absence of any given specific policy. This model shows a sub-optimal solution regarding the collective interest. In the economic sense, the traffic crowding represents an externality for the society.
- The *collective sub-model* examines the modified commuter behaviour, assuming that a toll has to be paid for road use during the peak time. This time-dependent toll is supposed to optimise the collective interest, by internalising the congestion externality observed in the individual model as discussed by Small (1983).

The basic idea behind both sub-models is that a car commuters chooses his (her) departure time from home to the office in the CBD by making a trade-off between two time aspects originally discussed by Small (1982):

- The *schedule time* imposed by the working hours in the CBD. The pressure imposed by schedule can be more or less dependent on the choice of the parameters in the model.
- The *travel time* imposed by the current traffic conditions on the road on the way to the CBD, assuming a unique departure point.

Accordingly, two cost functions are introduced for both time aspects. They represent penalties for each individual commuter. A potential toll charged during the peak hours gives an additional penalty. Commuter will make a trade-off between the cost components, including the possible peak-hour toll. The final result is to achieve a constant total marginal cost across the time board. In this way a unique distribution of departure times from home can be derived.

More formally, starting first with the individual sub-model, i.e. without considering a toll, assume that N commuters move to the CBD, and that the recommended arrival time is given by  $s_0$ . Mirabel (1997) gives the private marginal cost  $C_{MC}$  of a car commuter in function of the time  $t$ :

$$C_{MC}(t) = \alpha T_{road} + \beta(s_0 - t) \quad \text{if } s_1 \leq t \leq s_0 \quad (1a)$$

$$C_{MC}(t) = \alpha T_{road} + \nu(t - s_0) \quad \text{if } s_0 \leq t \leq s_2 \quad (1b)$$

$s_1$  is the arrival time of the earliest commuter [min.]

$s_2$  is the arrival time of the latest commuter [min.]

$\alpha$  is the unit cost associated to the travel time [arbitrary currency units/min.]

$\beta$  is the unit cost associated with the arrival time before the recommended time  $s_0$  [arbitrary currency units/min.]

$\nu$  is the unit cost associated with the arrival time after the recommended time  $s_0$  [arbitrary currency units/min.]

$T_{road}$  is the time spent on road [min.] given by

$$T_{road} = t_{min} + t_{congestion} * \left[ \frac{A(t)}{R} \right]^\gamma \quad (2)$$

$t_{min}$  is the minimum commuting time achieved in  $s_1, s_2$  [min.]

$t_{congestion}$  is the congestion time achieved when the arrival rate becomes equal to the nominal road capacity [min.]

$A(t)$  is the commuters' arrival rate at time  $t$  [cars/min.]

$R$  is the nominal capacity of the lane [cars/min.]

$\gamma$  is the elasticity of the travel time with respect to the commuters' arrival rate [-]

The three following constraints are considered :

- All N cars arrive between  $s_1$  and  $s_2$ .
- There is no congestion in  $s_1$  and  $s_2$ .
- The private marginal cost  $C_{MC}(t)$  (1a or 1b) is constant at each time  $t$ .

This is sufficient to determine a unique equilibrium distribution of arrival times  $A(t)$  without having to model explicitly the preferences of individual commuters.

The collective sub-model minimises the total commuters' cost. The latter is calculated by weighting the marginal costs in time ( $s$ ) given by (1a) and (1b) with the commuters' rate  $A(s)$  and integrating it over the complete time period between  $s_1$  and  $s_2$ . The minimisation under constraints gives an optimal control problem to be solved with the Euler's equations. The first order optimality conditions give a multiplier to be interpreted as a shadow price. This price represents the marginal social cost as being the sum of the private cost supported by each commuter and the cost imposed to all drivers due to the arrivals in time ( $s$ ). To achieve optimality the drivers arriving at this time must support this collective cost appearing in ( $s$ ). This time-dependant toll has cost units. It is given by the following equation:

$$Toll(s) = \alpha t_{congestion} \gamma \frac{[A(s)]^2}{R} \quad \text{for } s_1 \leq s \leq s_2, \quad (3)$$

The toll internalises the cost of congestion by applying a charge during the peak-hour period  $[s_1, s_2]$ . Regarding the effect of the toll on traffic the very important following result is obtained. The toll, while internalising the social cost of the sub-optimal individual solution, spreads out the peak-hour time, and therefore it significantly reduces the congestion amplitude expressed by the value of  $T_{road}$ . (see O.C.D.E. (1994) for a description of the expected effects of this public policy; the experience in Singapore is described in Wilson (1988)).

These results are shown in the following four scenarios, numbered 1 to 4 in all presentation of results:

1. No toll imposition during peak-hours and relatively inflexible working time, i.e. high penalty in case of late arrival time.
2. Toll imposition during peak-hours and relatively inflexible working time, i.e. high penalty in case of late arrival time.
3. No Toll imposition during peak-hours and relatively flexible working time, i.e. low penalty in case of late arrival time.
4. Toll imposition during peak-hours and relatively flexible working time, i.e. low penalty in case of late arrival time.

For deriving the numerical results, the following numerical assumptions have been made regarding the values of parameters in Small's model:

There are  $N = 10^5$  commuters. The chosen departure time lies between 6 a.m. and 12 a.m., i.e. between 1 and 360 minutes. The minimum travel time is  $t_{min} = 15$  minutes. Recommended arrival time in office is 9 a.m., i.e. at time  $t = s_0 = 180$  min. The dimensionless elasticity between the arrival rate and the congestion time is  $\gamma = 1.4$ . Therefore one has the following values of parameters:

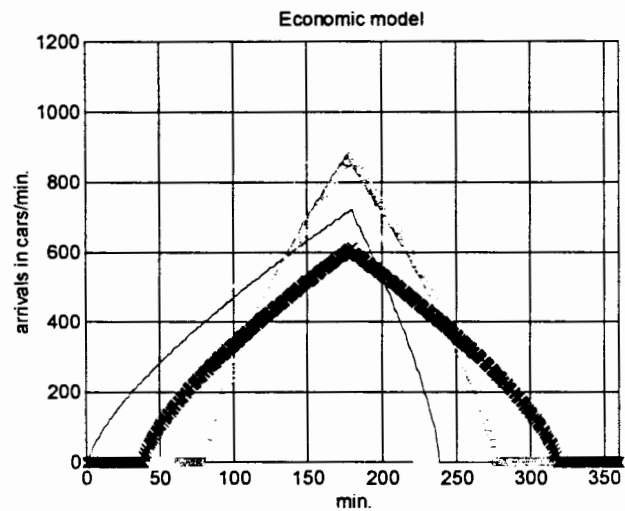
$$s_0 = 180 \text{ [min]}$$

$t_{\min}=15$  [min]  
 $\gamma=1.4$  [-]

The cost parameters are chosen to be:

$\alpha=1$  [currency unit/min]  
 $\beta=0.2$  [currency unit/min]  
 $\nu=0.6$  [currency unit/min] for the "inflexible" working hours scenario  
 $\nu=0.2$  [currency unit/min] for the "flexible" working hours scenario

Figures 1 and 2 show the results of the four scenarios for the arrival rate in [cars/min] and the congestion time above the minimum travel time, i.e.  $(T_{\text{road}} - t_{\min})$  in [min.]. Calculations have been made with MATLAB®. Figure 4 shows the computed toll value charged during the peak hours  $[s_1, s_2]$  for scenarios 2 and 4.



**Figure 1.** Arrival distribution rate in the economic model: scenarios 1 reference case (continuous thin line), scenario 2 use of toll (- - line), scenario 3 flexible working time (o), scenario 4 combined policy (x).

As to be expected, the influence of the toll is to spread out the relatively narrow time window of departures in the reference scenario 1. Of course this mechanism is largely enhanced towards later arrival times when combining the urban toll with more flexible working hours in scenario 4.

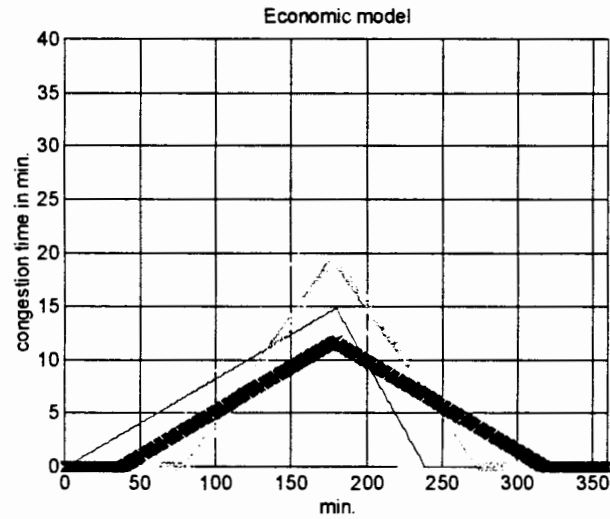
### 3. The commuters' preference model

In several respects the economic model described in the previous section is not entirely satisfactory.

1. The commuter is regarded as a "Homo oeconomicus". He (or she) keeps the marginal cost function constant throughout the whole time interval. In this approach, the evaluation of preferences with respect to a particular departure time is neither explicit nor transparent when

computing the marginal cost functions (1a) and (1b). Note also that for reasons of mathematical convenience, reference is always taken to arrival times rather than to departure times in the economic model as apparent in figure 1. In the Real World arbitrage between multiple incentive factors also takes place but in a less artificial way. The departure time imposing some more or less early wake-up hour, the waiting time in congestion, and penalties for late arrival time are mainly considered by commuters.

2. Aggregating costs in the same arbitrary cost units is a sensible thing to do, but the enumerated incentives are not conveniently translated into equivalent money values, as in the economic theory. For commuters each aspect has its value. It is often subjective and units or evaluation scales do not have homogeneous units. For example the inconvenience of getting up early in the morning will hardly be appreciated as a monetary cost by most people.
3. The model is inherently static. The equilibrium distribution of departures is established instantaneously as a mathematical artefact. In practice, the equilibrium distribution will result from trial and error, as experience of the road conditions in previous days will guide the commuters in making adjustments in their selection of a "best departure time".



**Figure 2.** Distribution of congestion times in the economic model: scenario 1 reference case (continuous thin line), scenario 2 use of toll (- - line), scenario 3 flexible working time (o), scenario 4 combined policy (x). The total time on road  $T_{road}$  is equal to  $t_{min} + \text{congestion time}$ .

Criticism 3 is made clear in the causal diagram of figure 3. It shows the dynamic congestion mechanism indicating the presence of feedback loops (Richardson and Pugh III, 1981). The proposed remedies to be sought for in urban toll and flexible working hours are also indicated. Although this diagram captures the snowball dynamics of congestion, it is essentially an open-loop model, as no feedback of experience flows from the previous-day-experience on the road.

In order to have a more transparent and dynamic modelling, the "economic model" has been adapted into a "preference model" now described.

At each time value (360 minutes in all) each commuter considers two elementary decisions: (a) to leave at this very time (b) to leave later. Of course this decisional process ought not to occur on the spot. It is rather the result of a mental scanning of each commuter in the previous evening to prepare the actual departure in the next morning. Also all available experience of the past traffic conditions is being accounted for during the process. The result is to prepare two time-dependent densities representing statistical preferences for departure times. The first density is positive  $\phi^+(t)$ ; it supports decision (a). The second density is negative  $\phi^-(t)$ ; it supports decision (b). The departure density  $\phi(t)$  will be chosen to be proportional to  $\phi^+(t)$ , appropriately normalised.

Both densities result themselves from evaluations to be conducted at the level of basic incentive factors, i.e. criteria, for selecting a particular departure time. In this respect the basic idea of Small's arbitrage is kept.

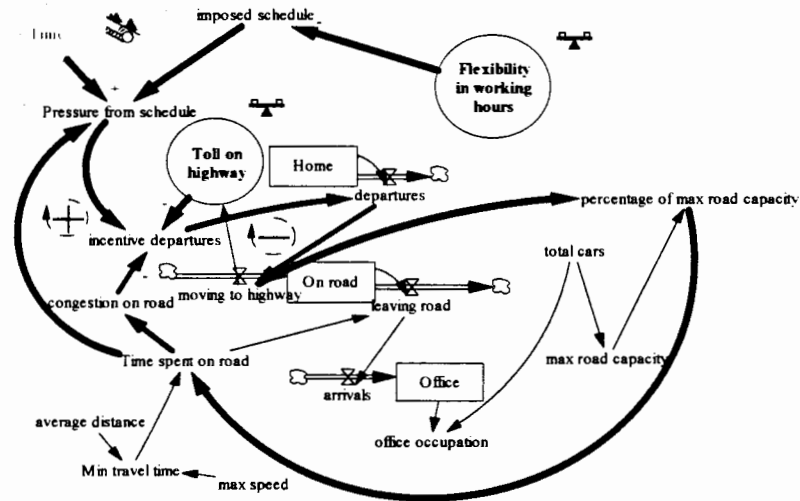
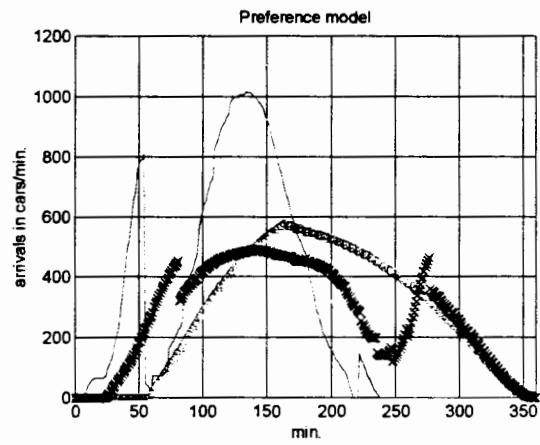


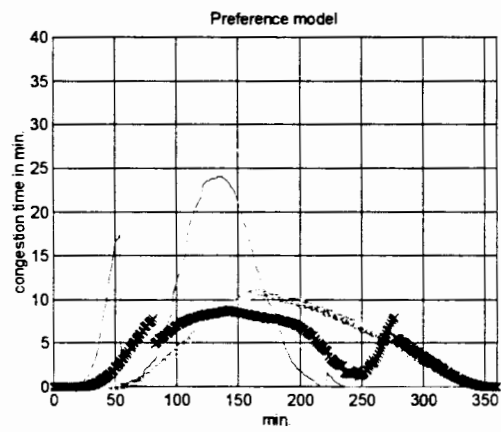
Figure 3. Influence diagram of the congestion mechanism on the commuting travel from home to office.

First, instead of considering unique marginal cost functions (1 a) and (1b) aggregating schedule time and travel time, well distinct criteria are established to represent preferences. Each criterion described below is established in its own relevant units, rather than in equivalent money values as in the marginal cost functions. In practice, only the relative changes in each criterion are important thanks to a normalisation process, as it will be shown below. Absolute units are therefore in fact arbitrary but it is important to keep coherence in units. Each criterion has to be minimised. It is assumed that the elementary time step is 1 min. In the present case each criterion  $C_i$  ( $i=1,4$ ) is evaluated minute per minute across the whole time interval 6 a.m. to 12 a.m., i.e. for 360 discrete values.

1. The first criterion represents the congestion time expressed in minutes, i.e. the difference between the travel time  $T_{road}$  and the minimum travel time  $t_{min}$ . The initial guess in each scenario



**Figure 6.** Arrival distribution in the preference model: scenario 1 reference case (continuous thin line), scenario 2 use of toll (- - line), scenario 3 flexible working time (o), scenario 4 combined policy (x).  
(results after 5 iterations to equilibrium).



**Figure 7.** Distribution of congestion times in the preference model: scenario 1 reference case (continuous thin line), scenario 2 use of toll (- - line), scenario 3 flexible working time (o), scenario 4 combined policy (x).  
(results after 5 iterations to equilibrium).

#### 4. Conclusion and further work

The paper has developed a "preference model" extending the "economic model" of K. Small applicable to congestion problems in urban area during the early rush hours from the green suburbs to the Central Business District. The main difference between the two models is that the aggregated cost function originally used in Small's model is split up into four preference criteria explicating the behaviours of commuters with respect to the particular choice of a departure time in the morning hours. Another important difference is that the congestion distributions of the preference model are obtained in the course of an iteration process, as commuters have to probe their collective behaviour before the equilibrium road conditions are obtained.

Assuming a reasonable choice of priority weights for the criteria gives congestion distributions, which are roughly comparable in shapes and amplitudes in both models. Comparing figures 2 (economic model) and 7 (preference model), a basic difference is observed in the scenario 2 (toll policy), however. The use of the urban toll without at the same time introducing more flexibility in working times seems to be largely an inefficient approach. This conclusion cannot be obtained with the economic model, but it is confirmed by common sense, as explained in the text. Flexibility alone is far superior to the sole use of the toll charge. The preferred solution seems to combine both strategies as it provides the least congestion most of the time. The merit of the proposed preference approach is to confirm this no doubt sensible result in a quantitative way.

Another observation is that the preference model gives the most comparable results to the economic model when the most importance in weights is given to the pressure of schedule, i.e. both undesirably early waking-up times and recommended arrival time. The inconvenience of increased travel times due to congestion is then of rather secondary importance for commuters. This result deserves to be further validated by actual observations and interrogation of commuters.

Note that many assumptions made in the preference model are not very satisfactory. Some could be easily relaxed like the assumption of linear preferences or the independence of the road conditions assumed for several preference criteria. In order to have a full model, showing the effects of flexibility, a full working day would have to be simulated, including the late afternoon congestion. This could easily be done. Another simplification of the model is to consider deterministic average-day conditions, i.e. ignoring disturbances in equilibrium arrival times caused by accidents, bad weather, etc. This might not be a too serious drawback as the idea is to demonstrate the overall efficiency of policies like the use of a toll or of flexible working times. It might nevertheless be a good idea to test the robustness of scenarios in presence of such stochastic disturbances.

Some criticisms are more serious, and could be the object of future research. First, the assumption of constant average preferences for all groups of commuters is quite restrictive. Arnott et al. (1992) discussed the appreciation of employees and executives with respect to the schedule time (see section 2) and evidenced profound behavioural differences between these two groups. Second, some thresholds must be taken into account to establish clear-cut preferences. For example leaving in the morning at 6 a.m. or at 6.02 a.m. will be indifferent for most commuters. These problems are addressed in preference-modelling techniques as used in Multicriteria Decision Aid (MCDA) (Brans et al., 1994). These techniques could be used for approaching in a more detailed way the four-criterion decision process of each individual commuter.

It is also planned to extend this approach to the analysis of further benefits induced by the urban toll for transforming the vicious circle of crowding into virtuous circles. for example promoting alternate transport means or reversing the trend of growing suburbs.

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