# *SOME METHODS FOR UPDATING INPUT-OUTPUT TABLES*

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## 1. *Introduction.*

One of the serious problems in regional analysis and forecasting is the lack of adequate and reliable data. Particularly, detailed interindustry models based on "technical coefficients", the so-called input-output models (I-O models), require a lot of information. It is altogether timeconsuming and costly to estimate a yearly I-O table. On the other hand, the technical coefficients once estimated are inherently unstable over a series of years. So, as is well known, the problem arises as to how to adjust a known I-O table on the basis of a limited quantity of information for later time periods.

One of the methods used to adjust a known I-O matrix from a certain basis year with the aid of known row and column totals from a later year is the so-called RAS-method. Expositions of this method are contained among others in Stone (1963), Paelinck and Waelbroeck (1963), Bacharach (1965), Schneider (1965), Theil (1966), Lecomber (1971), Glattfelder and Váczi (1972), Mazys (1972) and Van Straelen (1972). The major part of these authors do not only discuss the properties of the RAS-method itself, but attempt to solve certain shortcomings of the RAS-method. In addition to the RAS-method some alternative methods of updating I-O tables are developed, viz. the statistical correction method developed by Tilanus (1965) and the linear programming method developed by Matuszewski et al. (1964). A comparison of the RAS-method with the statistical correction method is contained in Tilanus (1965), while a comparison of the RAS-method with a linear programming method can be found in Schneider (1965).

In this paper attention will be paid to an alternative way of adjusting technical coefficients, viz. *a quadratic programming* method. An analytical expression for the updating procedure will be derived, and next the quadratic programming approach will be tested for I-O data of the Belgian economy. The results will be compared with the RAS-results obtained for the same data of the Belgian economy by inspecting the standard errors of the projections. For that reason first a brief exposition of the RAS-method will be presented.

## 2. The RAS-method.

The RAS-method or biproportional method of updating I-O matrices attempts to gauge simultaneously two effects in the adjustment procedure, viz. (1) relative shifts in the required input proportions of a certain activity (i.e., substitution), and (2) changes in the productivity (i.e., less inputs per unit of output). Both effects are assumed to exert a systematic *uniform* influence upon the rows and columns of I-O tables. The substitution effect requires a systematic adaptation of the *rows* of an I-O table, while the productivity effect requires a systematic adaptation of the successive *columns* of an I-O table. A uniform adjustment of the rows is obtained by premultiplying the I-O matrix with a diagonal matrix $\hat{r}$, while a uniform adjustment of the columns is obtained by postmultiplying the I-O matrix with a diagonal matrix $\hat{s}$. Therefore, a new I-O matrix $A^*$ is related to an I-O matrix A from a previous period as :

$$A^* = \hat{r}\, A\, \hat{s} \qquad (2.1)$$

The previous adjustment is possible only if $\hat{r}$ and $\hat{s}$ are known. The estimation of $\hat{r}$ and $\hat{s}$ for a certain year is based on the row and column totals of the year concerned. The following row and column data are necessary : the vector of sectoral production levels ($\underline{x}$), the vector of primary inputs per sector ($\underline{v}$), and the vector of final demand per sector ($\underline{f}$). By means of these data the total intermediate output of commodities ($\underline{u}$) and the total intermediate input into commodities ($\underline{y}$) can be calculated, respectively, as :

$$\underline{u} = \underline{x} - \underline{f} \qquad (2.2)$$

and :

$$\underline{y} = \underline{x} - \underline{v} \qquad (2.3)$$

By making use of the balance equation for supply and demand in the classical I-O model, viz.,

$$\underline{x} = A^* \underline{x} + \underline{f} \qquad (2.4)$$

it can easily be derived, that

$$\underline{u} = A^* \underline{x} \qquad (2.5)$$

Analogously, with the aid of the balance equation for production value and factor costs, viz.,

$$\underline{x} = \hat{x}(A^*)' \underline{i} + \underline{v} \; , \qquad (2.6)$$

one can derive that :

$$\underline{y} = \hat{x}(A^*)' \, \underline{i} \,, \tag{2.7}$$

where $\hat{x}$ is a diagonal matrix the diagonal elements of which are the elements of $\underline{x}$, and where $\underline{i}$ is a unit (summation) vector.

Substitution of (2.1) into (2.5) and (2.7) yields :

$$\hat{r} \, A \, \hat{s} \, \underline{x} = \underline{u} \tag{2.8}$$

and

$$\hat{x} \, \hat{s} \, A' \, \underline{r} = \underline{y} \,, \tag{2.9}$$

where $\underline{r}$ is a vector containing the diagonal elements of $\hat{r}$. The systems (2.8) and (2.9) are a set of nonlinear equations containing the unknown elements of $\hat{r}$ and $\hat{s}$. Since the number of equations is equal to the number of unknown elements, this system can, in principle, be solved.

The solution procedure itself is an iterative method converging towards the solution in a series of successive steps. The initial step is to insert into (2.8) the unit matrix as a preliminary solution for $\hat{s}$ and next to solve for the resulting value of $\hat{r}$. Then, the latter value is substituted into (2.9) in order to determine a new value for $\hat{s}$. Once this value has been calculated, one switches again to (2.8) in order to derive a new value for $\hat{r}$ and so forth, until the final solution is approximated up to a required degree of precision. The convergence and uniqueness of this RAS-procedure are discussed by Bacharach (1965).

It is obvious that the RAS-method is based on some rigorous assumptions in particular the assumption of a *uniform* effect over each column and over each row. As a counter example, Paelinck and Waelbroeck (1963) observed a bad estimation of the substitution effect in the case of the coal industry, since coal was used as a raw material in the coke industry and as a fuel input elsewhere. By eliminating *a priori* these elements from the RAS-procedure and by making an independent estimation of these elements the RAS-procedure can be applied to the remaining elements, taking into account the prior information concerning the previous elements. In general, by means of prior information the quality of the adjustments is considerably enlarged (so-called "truncated" RAS-method).

### 3. *Programming Methods for Updating I-O Tables.*

As mentioned, another method of updating the technical coefficients of an I-O matrix was a linear programming method developed by Matuszewski et al. (1964). This method minimizes the relative deviations between the original value and the adjusted value of the coefficients of an I-O table. If the coefficients of the original matrix A and of the updated matrix $A^*$ are denoted by $a_{ji}$ and $a^*_{ji}$, (j = 1, ...., I; i = 1, ..., I), respectively, the minimand is :

$$\min \omega = \sum_{i,j} \left| \frac{a_{ji} - a^*_{ji}}{a_{ji}} \right| \tag{3.1}$$

This objective function has to be minimized subject to the conditions (2.5) and (2.7); the result is essentially a linear programming model, which can be solved by means of standard techniques.

One of the major drawbacks of the linear programming method is the fact that the results may yield *negative* values for the updated coefficients. By introducing additional constraints, viz. lower limits of $a^*_{ji}$, non-negativity can be preserved. Another drawback of the linear programming method is its implicit rigidity : the solutions of the linear programming model are always *corner* solutions. This implies that frequently zero-values will be found for the adjusted coefficients, unless lower limits are imposed *a priori* (these lower limits are frequently rather arbitrary). An excessive positive variation of the coefficients can be prevented in a similar way by imposing arbitrary upper limits on the individual elements. Furthermore, corner solutions are often rather rigid with respect to minor changes in the minimand, so that often in the case of (3.1) a small shift in $a_{ji}$ will exert no influence at all. It was shown by Schneider (1965) that linear programming methods tend to provide adjustments with a lower quality than those of the RAS-method.

For that reason in this paper an alternative method of adjusting I-O coefficients will be derived, viz. a quadratic programming method. This method is less rigid than the linear programming method, and was first proposed by Friedlander (1961) for demographic projections. The purpose of this paper is to derive an *analytical* expression for the adjusted coefficients, and to compare the quality of the adjustments with those obtained by a RAS-method. In a next paragraph the quadratic programming method will be set out in more detail.

## 4. A Quadratic Programming Approach for the Adjustment of I-O Coefficients.

Instead of the objective function (3.1) it will be assumed here that the quadratic deviations between the original values and adjusted values of the I-O coefficients are to minimized. In order to prevent excessive variations in smaller coefficients the *relative* quadratic deviations are minimized. Therefore, the following quadratic programming (Q.P.) model arises (taking account of (2.5) and (2.7)) :

$$\begin{cases} \min \omega = \frac{1}{2} \sum_{i,j} \left( \frac{a_{ji} - a^*_{ji}}{a_{ji}} \right)^2 \\ \text{s.t.} \\ \underline{u} = A^* \underline{x} \\ \underline{y} = \hat{x} (A^*)' \underline{i} \end{cases} \tag{4.1}$$

Next, one may define :

$$a^*_{ji} = a_{ji} + \Delta_{ji} \tag{4.2}$$

or :

$$A^* = A + \Delta \tag{4.3}$$

Substitution of (4.2) and (4.3) into (4.1) yields :

$$\begin{cases} \min \omega = \frac{1}{2} \sum_{i,j} \left( \frac{\Delta_{ji}}{a_{ji}} \right)^2 \\ \text{s.t.} \\ \underline{u} = (A + \Delta) \underline{x} \\ \underline{y} = \hat{x} (A + \Delta)' \underline{i} \end{cases} \tag{4.4}$$

By defining :

$$\underline{du} = \underline{u} - A\underline{x} \tag{4.5}$$

and :

$$\underline{dy} = \underline{y} - \hat{x}A' \underline{i} \tag{4.6}$$

one obtains instead of (4.4) :

$$\begin{cases} \min \omega = \frac{1}{2} \sum_{i,j} \left( \frac{\Delta_{ji}}{a_{ji}} \right)^2 \\ \text{s.t.} \\ \underline{du} = \Delta\underline{x} \\ \underline{dy} = \hat{x} \Delta' \underline{i} \end{cases} \tag{4.7}$$

The Lagrangean function L associated with (4.7) is :

$$L = \frac{1}{2} \sum_{i,j} \left( \frac{\Delta_{ji}}{a_{ji}} \right)^2 - \underline{\lambda}' (\Delta \underline{x} - \underline{du}) - \underline{\mu}' (\hat{x} \Delta' \underline{i} - \underline{dy}) , \tag{4.8}$$

where $\underline{\lambda}$ and $\underline{\mu}$ are vectors of Lagrange multipliers associated with the previous systems of balance conditions. The Lagrangean function can be written in terms of individual elements as :

$$L = \frac{1}{2} \sum_{i,j} \left(\frac{\Delta_{ji}}{a_{ji}}\right)^2 - \sum_{i,j} \lambda_j \Delta_{ji} x_i + \sum_j \lambda_j \, du_j - \sum_{i,j} \mu_i \Delta_{ji} x_i + \sum_i \mu_i \, dy_i \qquad (4.9)$$

The first-order conditions associated with (4.9) are :

$$\left\{\begin{array}{lll}
\dfrac{\partial L}{\partial \Delta_{ji}} = \dfrac{\Delta_{ji}}{a_{ji}^2} - \lambda_j x_i - \mu_i x_i = 0 & \forall\, i,\ \forall\, j & \qquad (4.10) \\[2ex]
\dfrac{\partial L}{\partial \lambda_j} = \sum_i \Delta_{ji} x_i - du_j = 0 & \forall\, j & \\[2ex]
\dfrac{\partial L}{\partial \lambda_i} = \sum_j \Delta_{ji} x_i - dy_i = 0 & \forall\, i &
\end{array}\right.$$

The second-order conditions for a minimum are obviously satisfied (a convex minimand defined on a convex set of side-conditions). The previous first-order conditions can be represented in matrix notation as :

$$\left[\begin{array}{c|c|c}
\begin{matrix}\frac{1}{a_{II}^2} & & \\ & \ddots & \\ & & \frac{1}{a_{II}^2}\end{matrix}
&
\begin{matrix}
\begin{matrix} x_1 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ x_I & 0 & \cdots & 0 \end{matrix} \\ \hline
\begin{matrix} 0 & x_1 & 0 \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & x_I & 0 \cdots & 0 \end{matrix} \\ \hline
\vdots \\ \hline
\begin{matrix} 0 & \cdots & 0 & x_1 \\ \vdots & & \vdots & \vdots \\ 0 & \cdots & 0 & x_I \end{matrix}
\end{matrix}
&
\begin{matrix}
\begin{matrix} x_1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & x_I \end{matrix} \\ \hline
\begin{matrix} x_1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & x_I \end{matrix} \\ \hline
\vdots \\ \hline
\begin{matrix} x_1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & x_I \end{matrix}
\end{matrix}
\\ \hline
\begin{matrix}
\begin{matrix} x_1 & \cdots & x_I \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{matrix} & \ddots & \begin{matrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ x_1 & \cdots & x_I \end{matrix} \\ \hline
\begin{matrix} x_1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & x_I \end{matrix} & \ddots & \begin{matrix} x_1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & x_I \end{matrix}
\end{matrix}
& \multicolumn{2}{c}{0}
\end{array}\right]
\begin{bmatrix} \Delta_{11} \\ \Delta_{12} \\ \vdots \\ \Delta_{1I} \\ \hline \vdots \\ \hline \Delta_{I1} \\ \Delta_{I2} \\ \vdots \\ \Delta_{II} \\ \hline -\lambda_1 \\ \vdots \\ -\lambda_I \\ \hline -\mu_1 \\ \vdots \\ -\mu_I \end{bmatrix}
\begin{bmatrix} 0 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ 0 \\ \hline du_1 \\ \vdots \\ du_I \\ \hline dy_1 \\ \vdots \\ dy_I \end{bmatrix}
\qquad (4.11)$$

The latter system contains $I^2 + 2I$ equations with $I^2$ unknown variables $\Delta_{ji}$, I unknown variables $\lambda_j$ and I unknown variables $\mu_i$. Such a linear system of equations can be solved in principle, if all equations are independent; in other words, if the matrix of coefficients is non-singular. There is, however, a strong dependency among the equations, associated the side-conditions, because the total change in intermediate production should be equal to the total change in intermediate requirements :

$$\sum_j du_j = \sum_i dy_i \tag{4.12}$$

or :

$$\underline{i}' \, \underline{du} = \underline{i}' \, \underline{dy} \tag{4.13}$$

This can formally be proved by substituting (4.5) and (4.6) into (4.13), viz.

$$\underline{i}' \, \underline{u} - \underline{i}' \, A\underline{x} = \underline{i}' \, \underline{y} - \underline{i}' \, \hat{x} \, A' \, \underline{i} \, , \tag{4.14}$$

and next by substituting (2.5) and (2.7) into (4.14) :

$$\underline{i}' \, A^* \, \underline{x} - \underline{i}' \, A \, \underline{x} = \underline{i}'\hat{x}(A^*)' \, \underline{i} - \underline{i}'\hat{x}A' \, \underline{i} \tag{4.15}$$

or :

$$\begin{aligned} \underline{i}' \, \Delta\underline{x} &= \underline{i}' \, \hat{x} \, \Delta' \, \underline{i} \\ &= \underline{i}' \, \widehat{\Delta x} \, \underline{i} \\ &= \underline{i}' \, \Delta\underline{x} \quad , \text{q.e.d.} \end{aligned} \tag{4.16}$$

Therefore, one of the equations for the side-conditions can be dropped from the coefficient matrix of (4.11). This implies that one row and one corresponding column can be eliminated, so that the order of the coefficient matrix becomes $(I^2 + 2I - 1) \times (I^2 + 2I - 1)$. The resulting system of equations can now be written in a condensed form as :

$$B \, \underline{z} = \underline{c} \tag{4.17}$$

The unknown vector $\underline{z}$ is of order $(I^2 + 2I - 1)$; it contains $I^2$ unknown elements $\Delta_{ji}$ and $2I - 1$ unknown elements $\lambda_j$ and $\mu_i$. In a similar way the elements of B and of $\underline{c}$ can be considered. Assuming B is non-singular, one can easily solve $\underline{z}$ as :

$$\underline{z} = B^{-1} \, \underline{c} \tag{4.18}$$

In order to reduce the computational work one may carry out a partition of the matrix B as follows :

$$B = \begin{array}{c} I^2 \\ 2I-1 \end{array}\left[\begin{array}{c|c} \hat{d} & M \\ \hline K & O \end{array}\right], \qquad \begin{array}{cc} I^2 & 2I-1 \end{array} \tag{4.19}$$

where $\hat{d}$ is the left upperpart of the matrix B, viz. the $(I^2 \times I^2)$ diagonal matrix with main-diagonal elements $\frac{1}{a_{ji}^2}$. K, M and the zero-matrix O are defined in a similar way.
It is easily seen, that $K = M'$, so that K can be replaced by M'. The inverse matrix of B can be written according to the partition of B as :

$$B^{-1} = \left[\begin{array}{c|c} N & P \\ \hline Q & R \end{array}\right] \tag{4.20}$$

Obviously, the matrices N, P, Q and R have to be determined such that the following condition (multiplicative form of the inverse) is satisfied :

$$\left[\begin{array}{c|c} \hat{d} & M \\ \hline M' & O \end{array}\right] \left[\begin{array}{c|c} N & P \\ \hline Q & R \end{array}\right] = \left[\begin{array}{c|c} I & O \\ \hline O & I \end{array}\right] \tag{4.21}$$

The latter system can be written, successively, as :

$$\hat{d}\,N + MQ = I \tag{4.22}$$

$$\hat{d}\,P + MR = O \tag{4.23}$$

$$M'N\ (+OQ) = O \tag{4.24}$$

$$M'P\ (+OR) = I \tag{4.25}$$

Premultiplication of (4.22) with $M'\hat{d}^{-1}$ gives :

$$M'N + M'\hat{d}^{-1}\,MQ = M'\hat{d}^{-1} \tag{4.26}$$

Substitution of (4.24) into (4.26) gives :

$$M'\hat{d}^{-1} MQ = M'\hat{d}^{-1} \tag{4.27}$$

or :

$$Q = (M'\hat{d}^{-1} M)^{-1} M'\hat{d}^{-1} \tag{4.28}$$

Substitution of (4.28) into (4.22) gives the following solution for N :

$$N = \hat{d}^{-1} - \hat{d}^{-1} M(M'\hat{d}^{-1} M)^{-1} M'\hat{d}^{-1} \tag{4.29}$$

By premultiplying (4.23) with $M'\hat{d}^{-1}$ one obtains :

$$M'P + M'\hat{d}^{-1}MR = 0 \tag{4.30}$$

Next, by substituting (4.25) into (4.30) the solution for R is :

$$R = -(M'\hat{d}^{-1}M)^{-1} \tag{4.31}$$

Finally, substitution of (4.31) into (4.23) leads to the following solution for P :

$$P = \hat{d}^{-1} M(M'\hat{d}^{-1}M)^{-1} \tag{4.32}$$

Therefore, the solution vector (4.18) can now be written as :

$$\begin{bmatrix} \Delta_{11} \\ \vdots \\ \hline -\lambda_1 \\ \vdots \\ \hline -\mu_1 \\ \vdots \end{bmatrix} = \left[\begin{array}{c|c} \hat{d}^{-1} - \hat{d}^{-1}M(M'\hat{d}^{-1}M)^{-1} M'\hat{d}^{-1} & \hat{d}^{-1}M(M'\hat{d}^{-1}M)^{-1} \\ \hline (M'\hat{d}^{-1}M)^{-1}M'\hat{d}^{-1} & -(M'\hat{d}^{-1}M)^{-1} \end{array}\right] \begin{bmatrix} 0 \\ \vdots \\ \hline du_1 \\ \vdots \\ \hline dy_1 \\ \vdots \end{bmatrix} \tag{4.33}$$

It can easily be derived from (4.33), that the solution vector for changes in the I-O coefficients is equal to :

$$\begin{bmatrix} \Delta_{11} \\ \vdots \\ \vdots \end{bmatrix} = \begin{bmatrix} \hat{d}^{-1}M(M'\hat{d}^{-1}M)^{-1} \end{bmatrix} \begin{bmatrix} du_1 \\ \vdots \\ \hline dy_1 \\ \vdots \end{bmatrix} \tag{4.34}$$

The latter result implies a considerable reduction of computational requirements. The matrix $\hat{d}$ is a diagonal one, so that its inverse can be calculated directly. The inverse matrix of $M'\hat{d}^{-1}M$ can be calculated rather rapidly, since its order is $(2I-1) \times (2I-1)$. This means a considerable reduction compared to the inverse of the original matrix B of order $(I^2 + 2I-1) \times (I^2 + 2I-1)$. Therefore, the result can be calculated on almost each computer.

In the case of prior information concerning one or more of the elements $\Delta_{ji}$ the same procedure can be applied, viz. by eliminating these known elements *a priori* from the minimization procedure. Finally, it should be noted that the possibility of negative elements $\Delta_{ji}$ can be prevented by imposing the condition :

$$\Delta_{ji} \geq -a_{ji} , \qquad (4.35)$$

albeit that in this case the previous efficient solution procedure cannot be applied, so that a more time-consuming algorithm for quadratic programming with inequalities has to be used. In the following paragraph the result obtained by means of the Q.P.-procedure will be compared with those obtained by means of a RAS-method.

## 5. *A Comparison of the RAS-method and the Q.P.-method.*

The two methods, discussed in par. 2 and par.4, will be compared on the basis of I-O data for Belgium. An earlier analysis of changes in Belgian I-O matrices was carried out by Paelinck and Waelbroeck (1963).
In this paragraph the same data will be handled and a comparison between the RAS-method and the Q.P. method will be made. In their article Paelinck and Waelbroeck presented an I-O matrix of 21 Belgian sectors, which was estimated both for the year 1959 (in constant prices). These I-O data are included in Table 1 and 2 of the Appendix, respectively.

The 1959 data for intermediate sectoral inputs ($\underline{y}$), for intermediate sectoral outputs ($\underline{u}$) and for sectoral production values ($\underline{x}$) are contained in Table 3 of the Appendix. On the basis of the 1953 I-O table and of the successive column and row totals from 1959 one may approximate the 'real' I-O matrix from 1959 by means of the 'updating-techniques' described previously.

The results, obtained by Paelinck and Waelbroeck by means of a RAS-procedure, are reprinted in Table 4 of the Appendix. Next, one may compare this 'updated' matrix for 1959 (i.e. Table 4) with the matrix actually estimated for 1959 (i.e. Table 2). In this way one may inspect the 'power' and accuracy of the RAS-technique.

In a similar way one may deal with the Q.P.-technique described in par.4. By applying the successive matrix operations a new set of 'updated' input-output-coefficients for 1959 was obtained. The results can be found in Table 5 of the Appendix. Our purpose is now (1) to analyse the predictive 'power' of this Q.P.-approach by comparing the updated coefficients with the actual coefficients, and (2) to compare the relative value of the Q.P.-technique and the RAS-technique mutually.

A first method of testing the accuracy of the updated coefficients is to calculate the relative mean deviation ('mean prediction error') between the updated and the actual coefficients in 1959, denoted by $a^{*}_{ji}$ and $a^{o}_{ji}$, respectively. Such a mean deviation m can be defined as :

$$m = \frac{\sum_{j,i} |a^{o}_{ji} - a^{*}_{ji}|}{\sum_{j,i} a^{o}_{ji}} \qquad (5.1)$$

If the estimates $a^{*}_{ji}$ fall in the neighbourhood of the real coefficients $a^{o}_{ji}$, the mean prediction error becomes very small. This mean deviation can be calculated both for the RAS-technique and for the Q.P.-technique, and its value is a measure for the relative reliability of the techniques used.
The successive values of m for the RAS-technique and for the Q.P.-technique appeared to be equal to 0.094 and 0.105. This overall indicator shows that the mean prediction error of the Q.P.-technique is slightly higher than that of the RAS-technique. This relatively small difference suggests that there is no considerable difference between both methods.

A more accurate and detailed conclusion can be drawn by inspecting the sectoral mean deviations, both per column and per row. The mean deviation per column i is defined as :

$$m_i = \frac{\sum_{j} |a^{o}_{ji} - a^{*}_{ji}|}{\sum_{j} a^{o}_{ji}} \qquad (5.2)$$

The latter measure is an indicator for the (in)accuracy of the I-O coefficients for the intermediate inputs into each sector i.

In a similar way one may define a mean deviation for each row j as :

$$m_j = \frac{\sum_i |a^o_{ji} - a^*_{ji}|}{\sum_i a^o_{ji}} , \qquad (5.3)$$

which indicates the relative (in)accuracy of the updated coefficients for the intermediate outputs from each sector j.

The sectoral results of $m_i$ and $m_j$ both for the RAS- and for the Q.P.-procedure are contained in Table 6 and 7 of the Appendix, respectively. These results confirm the previous provisional conclusion, that there is no considerable difference in the Q.P.- and the RAS-results. In these sectoral outcomes, too, there is a slight tendency for the mean prediction error of the Q.P.-method to be somewhat higher than that of the RAS-technique, viz. in 15 cases of the column deviations $m_i$ ($i = 1, ..., 21$) and in 12 cases of the row deviations $m_j$ ($j = 1, ..., 21$).

In addition to a mean deviation one may inspect a relative quadratic deviation ('mean square prediction error') between the updated and the real coefficients. Such a relative quadratic deviation d can according to Theil (1966) be defined as :

$$d = \frac{\sum_{i,j} (a^o_{ji} - a^*_{ji})^2}{n} \qquad (5.4)$$

By taking the root of (5.4) one obtains the 'root-mean-square-prediction-error'. An alternative measure for the accuracy of the updating-technique is the so-called inequality coefficient q, defined as :

$$q = \frac{\sum_{j,i} (a^o_{ji} - a^*_{ji})^2}{\sum_{j,i} a^{o^2}_{ji}} \qquad (5.5)$$

It is easily seen that $0 \leq q \leq 1$, when $0 \leq a^*_{ji} \leq 2a^o_{ji}$. Furthermore, it is obvious that $q \rightarrow 0$, when $a^*_{ji} \rightarrow a^o_{ji}$ .

It is obvious that the latter measure bears some resemblance to the objective function of the Q.P.-procedure; this measure as well as the mean square predection error gives a higher (i.e., quadratic) 'penalty' to relatively higher deviations from the actual pattern. The values of q for the RAS-method and for the Q.P.-method are equal to 0.008 and 0.007, respectively. It appears that both values have a similar order of magnitude, albeit that now the Q.P.-method gives a slightly better result than the RAS-method. This confirms once more the provisional conclusion that there is no significant difference in the quality of the Q.P.-method and the RAS-method.

It is evident that for each column and for each row separately also a relative quadratic deviation can be calculated. Analogously to (5.2) and (5.3) one may define :

$$q_i = \frac{\sum_j (a^o_{ji} - a^*_{ji})^2}{\sum_j a^{o\,2}_{ji}} \qquad (5.6)$$

and :

$$q_j = \frac{\sum_i (a^o_{ji} - a^*_{ji})^2}{\sum_i a^{o\,2}_{ji}} \qquad (5.7)$$

The results of $q_i$ and $q_j$ both for the RAS-technique and for the Q.P.-technique are contained in Table 8 and 9 of the Appendix, respectively. These values show a global pattern nearly similar to that of $m_i$ and $m_j$. There are only slight variations among the $q_i$'s and $q_j$'s, and there is no significant difference in the inequality coefficients neither for the columns nor for the rows. It appears that in 10 cases the relative quadratic deviation $q_i$ ($i = 1, \ldots, 21$) of the Q.P.-method is lower than that of the RAS-method, whereas the corresponding row indicator $q_j$ ($j = 1, \ldots, 21$) is in 9 cases lower. Preliminarily, these results do not permit a firm conclusion in favour of one of both techniques.

An alternative way of obtaining more insight into the behaviour of the RAS-technique and of the Q.P.-technique is to carry out successively a regression analysis between the updated and the actual coefficients. By abandoning the intercept one obtains a regression line through the origin, the slope of which indicates whether the updating-technique concerned under- or overestimates the actual coefficients. The values of the regression coefficient for the RAS- and the Q.P.-technique appear to be equal to 0.954 and 0.962, respectively, whereas the successive standard errors of estimation are 0.008 and 0.004. The values of the regression coefficients show only a minor difference, so that these results are not suitable to discriminate between one of both techniques.

In addition, one can calculate the correlation coefficient associated with the previous regression procedure. The value of this correlation coefficient, denoted by r, indicates the degree to which there is a linear correlation between the updated and the actual coefficients; in other words, the degree to which there exists a systematic scalter diagram between $a_{ji}^{o}$ and $a_{ji}^{*}$, which shows a close linear relationship through the origin between $a_{ji}^{o}$ and $a_{ji}^{*}$. The successive values of r for the RAS- and the Q.P.-technique appear to be 0.996 and 0.997. These results again lead to the conclusion that on the average the RAS- and the Q.P.-technique have about the same 'power' of updating or predicting I-O tables.

It is obvious that a similar procedure can be applied to each row and column separately. The regression coefficients as well as the correlation coefficients for all individual columns and rows can be defined in a similar way, viz. for each column i and for each row j. Their values are contained in Table 10 and 11 of the Appendix. Although there are some variations among the regression coefficients of the RAS- and of the Q.P.-technique, the results show globally a similar pattern. In both cases the regression coefficients are approximately equal to 1, so that both methods appear to provide good estimates of the actual pattern, although there are differences of a minor order. The successive corresponding correlation coefficients show in general a same order of magnitude, so that more and more the conclusion is justified, that both techniques possess the same quality in updating and predicting I-O coefficients.

Finally, one may inspect whether there is a systematic relationship between the absolute difference in the actual and the updated coefficients (i.e., the prediction error) on the one hand, and the actual coefficients themselves on the other hand. So, by defining :

$$\Delta a_{ji} = |a_{ji}^{o} - a_{ji}^{*}| , \qquad (5.8)$$

one may check whether there is a systematic linear relationship between $\Delta a_{ji}$ and $a_{ji}^{o}$, both for the RAS-method and for the Q.P.-method. An (arbitrary) example of such a linear relationship is contained in figure 1.

The value of the regression coefficient between $\Delta a_{ji}$ and $a_{ji}^{o}$ indicates whether there is a systematic link between the absolute value of the prediction error per element and the value of the coefficient itself. These regression coefficients appeared to be equal to 0.071 and 0.058 for the RAS- and the Q.P.-method, respectively, with respective standard errors 0.003 and 0.003. This result indicates once more that a definite conclusion in favour of one of both techniques is hard to draw, as appears also from the correlation coefficients, which are equal to 0.754 and 0.621, respectively.

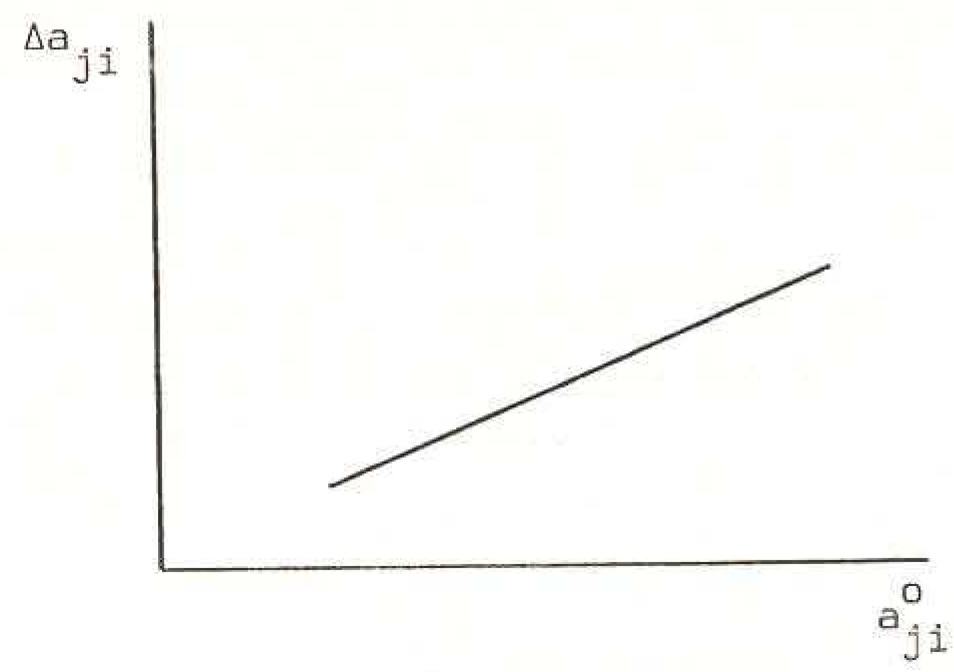


Figure 1. A linear relationship between prediction errors and actual values of I-O coefficients.

In a similar way one may calculate the regression coefficients and the correlation coefficients per row and per column both for the RAS- and the Q.P.-method. These coefficients can be found in Table 12 and 13 of the Appendix, respectively.

The values of these coefficients appear to show considerable differences both per row and per column. There is, however, no systematic discriminating link either with respect to the individual sectors concerned or with respect to columns and rows. The only conclusion that can be drawn is that the size of the prediction error shows only a slightly positive relationship with respect to the level of the actual coefficient, both for the RAS- and for the Q.P.-technique.

By including an intercept into the previous regression procedure, the results were not essentially affected, except that the constant term itself appeared to be frequently non-significant.

## *6. Evaluation and Outline of Further Applications.*

The results presented in par. 5 show that a unique conclusion either in favour of the RAS-technique or in favour of the Q.P.-technique cannot be drawn. The differences in the mean prediction errors, in the mean square errors and in the regression and correlation coefficients do not permit a definite conclusion. The specific property of the Q.P.-method is that it tends to truncate large deviations from an initial value of a coefficient owing to the quadratic 'penalty' function, as can be illustrated by inspecting the results for the elements $a_{23}$ and $a_{43}$, both for the RAS- and for the Q.P.-technique. This suggests that the Q.P.-method might be helpful in the case of short-run adaptations of I-O tables, when considerable changes in I-O coefficients are less acceptable. Preliminarily, the general conclusion may be that the RAS-technique and the Q.P.-technique are almost equivalent methods in updating problems.

Finally, it should be noted that the previous updating techniques can not only be applied in the case of I-O models, but also in demographic, traffic and modal-split models, and in many other allocation models which suffer from a lack of reliable and adequate permanent information.

A very specific application of the techniques described previously might be in pollution problems. As is well known, the relationship between the emission of pollution and the level of production can be represented by means of constant pollution-I-O-coefficients (P-I-O-coefficients). Such a relationship can be represented as :

$$\underline{e} = B\underline{x} , \tag{6.1}$$

where $\underline{e}$ represents a vector of order (K x 1) with elements $e_k$ (k=1, - , k), representing the level of emission of pollutant k (carbon monoxide, sulfur dioxide, etc.). The relationship between the level $e_k$ of pollutant k and the level $x_i$ of production i (i = 1, ..., I) can be represented by means of the P-I-O-coefficient $b_{ki}$. The matrix B in (6.1), of order K x I, contains all these P-I-O-coefficients.

A very serious problem in pollution research is the estimation of the matrix B, as well as the yearly updating of this matrix. This matrix appears to be rather unstable, because the large-scale environmental deterioration forces entrepreneurs to implement alternative production processes leading to considerable changes both in the volume and in the 'mix' of emitted pollutants. It is extremely difficult to collect yearly data for these technical changes.

Therefore, an alternative approach might be to estimate the changes in B with the aid of known marginal data, given a known value of B in a certain basis year. Then the only problem is to collect data for $\underline{e}$ and $\underline{x}$. In general, it will be possible to estimate the sectoral production levels $\underline{x}$, but the determination of the emission of pollution $\underline{p}$ is frequently overloaded with difficulties.

In this case an alternative way may be to approximate the volume of emitted pollutants by means of data concerning the *concentration* of pollution at several observation points. In taking account of wind speed, wind direction and meteorological stability conditions the concentration of pollution at certain points can be 'transformed' into estimated emission values at the source by means of meteorological diffusion formulae. Such a diffusion analysis enables one to approximate the average emission of pollution at a certain region, given a series of measurements on concentrations of pollution at several points (inside and/or outside the region). Once the unknown level of p has been estimated, one could use (6.1) as a side-condition in the Q.P.-technique, viz. by minimizing the relative quadratic differences between actual and past values of the P-I-O-coefficients, given the new marginal conditions (6.1).

The previous procedure shows still another possibility of the Q.P.-technique. It serves to solve the number of degrees of freedom in updating problems without any restrictions on the number of side-conditions; for instance, in the previous P-I-O-case only *horizontal* additivity conditions were imposed, while the vertical conditions are disregarded. In this case the classical RAS-technique is not applicable, since it is based on horizontal and vertical marginal data, though a simplified RA- or AS-technique might give a first approximation. Whether Q.P.- or RAS-method is better in this case has still to be investigated.

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Table 1. Input-Output-Matrix from 1953

| | sectors | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | oil | - | .006 | .001 | .003 | .002 | .011 | .006 | .025 | .009 | .009 | .007 | .007 | .018 | .010 | .007 | .035 | .008 | .010 | .005 | - | - |
| 2 | coal | .001 | - | .676 | .232 | .003 | .024 | .004 | .032 | .014 | .041 | .003 | .010 | .077 | - | .005 | .010 | .003 | .004 | .006 | - | - |
| 3 | coke and gas | - | .002 | - | .002 | - | .051 | - | - | .188 | .028 | .006 | .001 | .006 | - | .001 | .001 | - | .001 | .001 | - | - |
| 4 | electricity | .004 | .021 | .015 | - | .012 | .036 | .003 | .012 | .011 | .012 | .010 | .013 | .024 | - | .013 | .012 | .006 | .003 | .006 | - | - |
| 5 | various industries | - | - | - | - | - | .026 | - | - | .035 | .174 | .012 | .012 | .006 | .002 | .007 | .002 | - | .001 | .002 | - | - |
| 6 | chemistry | .027 | .013 | .004 | .002 | .011 | - | .001 | .009 | .002 | .008 | .026 | .032 | .029 | .012 | .023 | .008 | .038 | .001 | .036 | - | - |
| 7 | finances | - | .007 | .001 | .002 | .001 | .003 | - | .009 | .004 | .002 | .005 | .004 | .004 | .006 | .003 | .020 | .005 | .002 | .004 | .018 | - |
| 8 | transport/ communication | .026 | .015 | .066 | .044 | .007 | .048 | .049 | - | .073 | .020 | .022 | .021 | .056 | .030 | .018 | .092 | .017 | .019 | .010 | - | - |
| 9 | iron and steel | - | .019 | .026 | - | .005 | - | - | .006 | - | - | .199 | - | .006 | .039 | - | - | .002 | - | - | - | - |
| 10 | non-ferrous metals | - | - | - | .009 | .018 | .035 | - | .002 | .008 | - | .043 | - | - | .026 | - | - | - | .001 | .001 | - | - |
| 11 | metal working | .005 | .017 | .011 | .020 | .052 | .007 | - | .044 | .017 | .009 | - | .022 | .013 | .043 | .007 | .002 | .004 | .005 | .003 | - | - |
| 12 | wood/paper | .005 | .029 | .001 | .014 | .008 | .046 | .030 | .007 | .004 | - | .022 | - | .024 | .062 | .015 | .036 | .005 | .015 | .009 | - | - |
| 13 | construction materials | - | .001 | .006 | .004 | .006 | .013 | - | .002 | .028 | .003 | .006 | .004 | - | .143 | .002 | .001 | .003 | .010 | .002 | - | - |
| 14 | construction | .001 | .008 | .002 | .007 | .041 | .005 | .001 | .006 | .002 | .004 | .004 | .005 | .007 | - | .006 | .007 | .007 | .003 | .002 | .118 | - |
| 15 | leather/textile | - | .002 | - | - | .007 | .016 | - | .001 | - | - | .006 | .003 | - | .001 | - | .001 | .002 | .002 | .003 | - | - |
| 16 | commerce | - | .001 | - | - | .004 | .004 | .002 | .002 | .002 | .002 | .002 | .011 | .009 | .004 | .007 | - | .034 | .009 | .008 | - | - |
| 17 | agriculture/ forestry/fishery | - | - | - | - | .002 | - | - | - | - | - | - | .044 | - | - | .013 | - | - | .285 | - | - | - |
| 18 | food | - | - | - | - | - | .022 | - | - | - | - | - | - | - | - | .004 | - | .046 | - | .120 | - | - |
| 19 | various services | .002 | .003 | - | .010 | - | .012 | .016 | .002 | .001 | - | .005 | .001 | .001 | .003 | .004 | .020 | .006 | .003 | - | .006 | - |
| 20 | hotel | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 21 | government | - | - | - | - | .025 | - | - | .001 | - | - | - | .039 | - | .003 | .001 | - | - | - | - | - | - |

Table 2. Input-Output-Matrix from 1959

| | sectors | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | oil | - | .005 | .003 | .011 | .004 | .016 | .008 | .028 | .011 | .017 | .007 | .009 | .030 | .009 | .003 | .037 | .009 | .011 | .007 | - | - |
| 2 | coal | .001 | - | .671 | .192 | .001 | .009 | .002 | .013 | .007 | .023 | .002 | .005 | .056 | - | .003 | .004 | .001 | .002 | .004 | - | - |
| 3 | coke and gas | - | .002 | - | .002 | - | .040 | - | - | .161 | .025 | .006 | .001 | .006 | - | .001 | .001 | - | .001 | .001 | - | - |
| 4 | electricity | .005 | .024 | .014 | - | .018 | .036 | .004 | .018 | .010 | .015 | .010 | .013 | .024 | - | .015 | .014 | .008 | .004 | .008 | - | - |
| 5 | various industries | - | - | - | - | - | .023 | - | - | .033 | .106 | .011 | .011 | .006 | .002 | .007 | .002 | - | .001 | .002 | - | - |
| 6 | chemistry | .036 | .014 | .005 | .002 | .015 | - | .001 | .011 | .002 | .009 | .031 | .035 | .033 | .016 | .031 | .010 | .052 | .003 | .057 | - | - |
| 7 | finances | - | .007 | .001 | .002 | .001 | .003 | - | .007 | .004 | .002 | .005 | .004 | .004 | .006 | .003 | .021 | .005 | .002 | .004 | .018 | - |
| 8 | transport/ communication | .032 | .017 | .072 | .044 | .008 | .045 | .048 | - | .083 | .028 | .023 | .024 | .055 | .032 | .019 | .092 | .018 | .019 | .011 | - | - |
| 9 | iron and steel | - | .018 | .026 | - | .005 | - | - | .005 | - | - | .205 | - | .006 | .039 | - | - | .002 | - | - | - | - |
| 10 | non-ferrous metals | - | - | - | .008 | .019 | .030 | - | .002 | .007 | - | .040 | - | - | .026 | - | - | - | .001 | .001 | - | - |
| 11 | metal working | .010 | .028 | .012 | .022 | .063 | .007 | - | .056 | .018 | .014 | - | .023 | .014 | .047 | .007 | .002 | .004 | .012 | .004 | - | - |
| 12 | wood/paper | .007 | .019 | .001 | .014 | .010 | .044 | .033 | .007 | .005 | - | .022 | - | .027 | .062 | .017 | .041 | .006 | .017 | .011 | - | - |
| 13 | construction materials | - | .001 | .006 | .004 | .007 | .011 | - | .002 | .027 | .003 | .006 | .004 | - | .143 | .002 | .001 | .003 | .010 | .002 | - | - |
| 14 | construction | .001 | .008 | .002 | .007 | .041 | .004 | .001 | .005 | .002 | .004 | .004 | .005 | .007 | - | .006 | .007 | .007 | .003 | .002 | .118 | - |
| 15 | leather/textile | - | .002 | - | - | .008 | .014 | - | .001 | - | - | .006 | .003 | - | .001 | - | .001 | .002 | .002 | .003 | - | - |
| 16 | commerce | - | .001 | - | - | .004 | .003 | .002 | .002 | .002 | .002 | .002 | .010 | .009 | .004 | .007 | - | .033 | .009 | .008 | - | - |
| 17 | agriculture/ forestry/fishery | - | - | - | - | .002 | - | - | - | - | - | - | .016 | - | - | .013 | - | - | .286 | - | - | - |
| 18 | food | - | - | - | - | - | .019 | - | - | - | - | - | - | - | - | .004 | - | .049 | - | .114 | - | - |
| 19 | various services | .002 | .003 | - | .010 | - | .010 | .016 | .002 | .001 | - | .005 | .001 | .001 | .003 | .004 | .021 | .006 | .003 | - | .006 | |
| 20 | hotel | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 21 | government | - | - | - | - | .027 | - | - | .001 | - | - | - | .032 | - | .003 | .004 | - | - | - | - | - | - |

Table 3. Column and Row Totals from 1959 ($10^9$ B.F.)

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\underline{y}$ | 1,40 | 2,76 | 9,48 | 4,22 | 4,78 | 8,52 | 1,64 | 8,80 | 13,40 | 3,27 | 29,96 | 4,49 | 6,02 | 25,57 | 7,37 | 17,00 | 11,61 | 37,71 | 19,50 | 6,66 | 0,00 |
| $\underline{u}$ | 10,08 | 14,41 | 8,16 | 8,44 | 5,36 | 17,00 | 4,94 | 24,07 | 19,82 | 6,72 | 12,59 | 15,88 | 13,00 | 9,36 | 1,83 | 5,05 | 29,00 | 12,78 | 3,93 | 0,00 | 1,74 |
| $\underline{x}$ | 14,72 | 18,52 | 11,66 | 13,26 | 20,49 | 27,14 | 14,27 | 55,05 | 35,88 | 13,18 | 77,92 | 22,89 | 21,64 | 64,99 | 48,78 | 66,93 | 56,70 | 97,82 | 81,72 | 46,82 | 53,19 |

Table 4. Updated Input-Output-Matrix for 1959
estimated by means of RAS-technique

| | sectors | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | oil | - | .007 | .002 | .004 | .002 | .011 | .007 | .028 | .010 | .009 | .008 | .007 | .021 | .011 | .009 | .040 | .009 | .013 | .006 | - | .204 |
| 2 | coal | .001 | - | .615 | .175 | .002 | .014 | .003 | .020 | .009 | .024 | .002 | .006 | .051 | - | .003 | .006 | .002 | .003 | .004 | - | .440 |
| 3 | coke and gas | - | .002 | - | .002 | - | .040 | - | - | .165 | .022 | .005 | .001 | .005 | - | .001 | .001 | - | .001 | .001 | - | .246 |
| 4 | electricity | .005 | .023 | .024 | - | .015 | .038 | .004 | .013 | .013 | .012 | .011 | .012 | .029 | - | .016 | .014 | .007 | .004 | .008 | - | .248 |
| 5 | various industries | - | - | - | - | - | .021 | - | - | .031 | .135 | .010 | .009 | .005 | .002 | .007 | .002 | - | .001 | .002 | - | .225 |
| 6 | chemistry | .041 | .016 | .007 | .003 | .015 | - | .001 | .011 | .003 | .009 | .033 | .034 | .039 | .015 | .032 | .010 | .053 | .002 | .052 | - | .376 |
| 7 | finances | - | .006 | .001 | .002 | .001 | .003 | - | .008 | .004 | .002 | .005 | .003 | .004 | .006 | .003 | .019 | .005 | .002 | .004 | .018 | .096 |
| 8 | transport/ communication | .031 | .014 | .094 | .052 | .008 | .044 | .050 | - | .074 | .018 | .022 | .018 | .059 | .030 | .019 | .093 | .018 | .022 | .011 | - | .677 |
| 9 | iron and steel | - | .019 | .038 | - | .006 | - | - | .006 | - | - | .202 | - | .006 | .040 | - | - | .002 | - | - | - | .319 |
| 10 | non-ferrous metals | - | - | - | .010 | .019 | .031 | - | .002 | .008 | - | .041 | - | - | .025 | - | - | - | .001 | .001 | - | .138 |
| 11 | metal working | .007 | .020 | .019 | .029 | .068 | .008 | - | .052 | .021 | .010 | - | .022 | .017 | .052 | .009 | .003 | .005 | .007 | .004 | - | .353 |
| 12 | wood/paper | .006 | .029 | .002 | .017 | .009 | .043 | .032 | .007 | .004 | - | .023 | - | .026 | .063 | .017 | .028 | .006 | .018 | .011 | - | .351 |
| 13 | construction materials | - | .001 | .008 | .005 | .006 | .012 | - | .002 | .028 | .003 | .006 | .003 | - | .140 | .002 | .001 | .003 | .011 | .002 | - | .233 |
| 14 | construction | .001 | .007 | .003 | .008 | .041 | .004 | .001 | .006 | .002 | .003 | .004 | .004 | .007 | - | .006 | .007 | .007 | .003 | .002 | .118 | .234 |
| 15 | leather/textile | - | .002 | - | - | .007 | .014 | - | .001 | - | - | .006 | .002 | - | .001 | - | .001 | .002 | .002 | .003 | - | .041 |
| 16 | commerce | - | .001 | - | - | .004 | .003 | .002 | .002 | .002 | .002 | .002 | .008 | .009 | .004 | .007 | - | .034 | .010 | .008 | - | .098 |
| 17 | agriculture/ forestry/fishery | - | - | - | - | .002 | - | - | - | - | - | - | .032 | - | - | .012 | - | - | .238 | - | - | .329 |
| 18 | food | - | - | - | - | - | .018 | - | - | - | - | - | - | - | - | .004 | - | .044 | - | .118 | - | .184 |
| 19 | various services | .002 | .003 | - | .011 | - | .011 | .016 | .002 | .001 | - | .005 | .001 | .001 | .003 | .004 | .019 | .006 | .003 | - | .006 | .094 |
| 20 | hotel | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 21 | government | - | - | - | - | .029 | - | - | .001 | - | - | - | .036 | - | .003 | .001 | - | - | - | - | - | .070 |

Table 5. Updated Input-Output-Matrix for 1959, estimated by means of Q.P.-technique[1)]

| | sectors | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | oil | - | .006 | .001 | .003 | .002 | .011 | .006 | .030 | .019 | .009 | .007 | .007 | .020 | .011 | .008 | .045 | .009 | .012 | .005 | - | - |
| 2 | coal | .001 | - | .647 | .185 | .003 | .014 | .004 | .006 | .011 | .029 | .003 | .008 | .040 | - | .005 | .007 | .003 | .003 | .005 | - | - |
| 3 | coke and gas | - | .001 | - | .002 | - | .039 | - | - | .161 | .027 | .006 | .001 | .006 | - | .001 | .001 | - | .001 | .001 | - | - |
| 4 | electricity | .004 | .023 | .017 | - | .013 | .041 | .003 | .014 | .012 | .012 | .012 | .013 | .030 | - | .016 | .015 | .007 | .003 | .007 | - | - |
| 5 | various industries | - | - | - | - | - | .022 | - | - | .031 | .122 | .010 | .011 | .006 | .002 | .007 | .002 | - | .001 | .002 | - | - |
| 6 | chemistry | .040 | .014 | .004 | .002 | .012 | - | .001 | .010 | .002 | .008 | .033 | .029 | .036 | .013 | .031 | .009 | .056 | .001 | .060 | - | - |
| 7 | finances | - | .007 | .001 | .002 | .001 | .003 | - | .009 | .004 | .002 | .005 | .004 | .004 | .006 | .003 | .018 | .005 | .002 | .004 | .019 | - |
| 8 | transport/ communication | .036 | .015 | .091 | .055 | .007 | .040 | .051 | - | .077 | .020 | .021 | .018 | .070 | .028 | .020 | .089 | .018 | .023 | .010 | - | - |
| 9 | iron and steel | - | .019 | .030 | - | .005 | - | - | .006 | - | - | .203 | - | .006 | .040 | - | - | .002 | - | - | - | - |
| 10 | non-ferrous metals | - | - | - | .009 | .019 | .031 | - | .002 | .008 | - | .040 | - | - | .025 | - | - | - | .001 | .001 | - | - |
| 11 | metal working | .005 | .118 | .012 | .023 | .070 | .007 | - | .062 | .019 | .009 | - | .020 | .014 | .055 | .008 | .002 | .004 | .006 | .003 | - | - |
| 12 | wood/paper | .005 | .030 | .001 | .015 | .008 | .041 | .031 | .007 | .004 | - | .022 | - | .027 | .064 | .017 | .039 | .005 | .019 | .010 | - | - |
| 13 | construction materials | - | .001 | .006 | .004 | .006 | .013 | - | .002 | .029 | .003 | .006 | .004 | - | .138 | .002 | .001 | .003 | .012 | .002 | - | - |
| 14 | construction | .001 | .008 | .002 | .007 | .043 | .005 | .001 | .006 | .002 | .004 | .004 | .005 | .007 | - | .006 | .007 | .007 | .003 | .002 | .012 | - |
| 15 | leather/textile | - | .002 | - | - | .007 | .015 | - | .001 | - | - | .006 | .003 | - | .001 | - | .001 | .002 | .002 | .003 | - | - |
| 16 | commerce | - | .001 | - | - | .004 | .004 | .002 | .002 | .002 | .002 | .002 | .010 | .009 | .004 | .007 | - | .033 | .009 | .008 | - | - |
| 17 | agriculture/ forestry/fishery | - | - | - | - | .002 | - | - | - | - | - | - | .026 | - | - | .013 | - | - | .284 | - | - | - |
| 18 | food | - | - | - | - | - | .019 | - | - | - | - | - | - | - | - | .004 | - | .046 | - | .116 | - | - |
| 19 | various services | .002 | .003 | - | .011 | - | .011 | .016 | .002 | .001 | - | .005 | .001 | .001 | .003 | .004 | .019 | .006 | .003 | - | .006 | - |
| 20 | hotel | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 21 | government | - | - | - | - | .030 | - | - | .001 | - | - | - | .035 | - | .003 | .001 | - | - | - | - | - | - |

1) The authors are indebted to Peter Mastenbroek, who carried out the computer analysis of the Q.P.-technique.

Table 6. Mean prediction Error of RAS-method[1]
per Column and per Row.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $m_i$ | .105 | .195 | .140 | .151 | .073 | .067 | .043 | .119 | .075 | .238 | .031 | .204 | .126 | .041 | .060 | .055 | .049 | .047 | .042 | - | - |
| $m_j$ | .209 | .100 | .036 | .150 | .183 | .080 | .051 | .109 | .062 | .045 | .175 | .076 | .047 | .026 | .047 | .041 | .063 | .054 | .043 | - | .134 |

Table 7. Mean Prediction Error of Q.P.-method
per Column and per Row.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $m_i$ | .160 | .221 | .065 | .091 | .116 | .105 | .087 | .163 | .062 | .210 | .039 | .209 | .176 | .071 | .060 | .106 | .082 | .174 | .063 | .014 | - |
| $m_j$ | .257 | .080 | .032 | .154 | .118 | .096 | .121 | .146 | .026 | .030 | .152 | .090 | .052 | .056 | .209 | .092 | .183 | .027 | .096 | - | .119 |

Table 8. Inequality Coefficient of RAS-method
per Column and per Row

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $q_i$ | .014 | .069 | .009 | .012 | .006 | .007 | .002 | .020 | .004 | .074 | - | .079 | .020 | .002 | .007 | .003 | .004 | .001 | .003 | - | - |
| $q_j$ | .057 | .007 | .001 | .041 | .065 | .008 | .007 | .023 | .004 | .002 | .025 | .011 | .001 | - | .006 | .004 | .003 | .003 | .006 | - | .016 |

Table 9. Inequality Coefficient of Q.P.-method
per Column and per Row

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $q_i$ | .022 | .057 | .002 | .006 | .013 | .015 | .004 | .025 | .002 | .032 | - | .051 | .063 | .004 | .006 | .008 | .006 | .029 | .001 | - | - |
| $q_j$ | .078 | .002 | - | .029 | .018 | .009 | .017 | .027 | - | .001 | .029 | .013 | .002 | .001 | .027 | .006 | .029 | .001 | .010 | - | .012 |

1) The authors are indebted to Peter Hamburger and Jos Wieleman, who carried out the computer analysis for the comparison of the RAS- and Q.P.-method.

Table 10. Regression Results between Updated and Actual Coefficients[1)] per Column and per Row for RAS-method.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Regression coefficient | 1.044 | .975 | .922 | .931 | 1.038 | .994 | 1.015 | .956 | .999 | 1.179 | .989 | 1.023 | .999 | .994 | 1.019 | 1.001 | .976 | .990 | 1.010 | .999 | - |
| per column i | (.025) | (.058) | (.011) | (.019) | (.015) | (.018) | (.009) | (.030) | (.013) | (.046) | (.004) | (.062) | (.031) | (.009) | (.019) | (.011) | (.014) | (.005) | (.011) | (.000) | - |
| Regression coefficient | .901 | .917 | 1.020 | 1.034 | 1.221 | 1.018 | .956 | 1.029 | .995 | 1.018 | 1.024 | 1.008 | .983 | .999 | .967 | 1.014 | .993 | 1.012 | .979 | - | 1.096 |
| per row j | (.048) | (.004) | (.005) | (.045) | (.028) | (.020) | (.015) | (.033) | (.013) | (.010) | (.035) | (.024) | (.005) | (.004) | (.016) | (.014) | (.013) | (.011) | (.016) | - | (.015) |
| Correlation coefficient per column i | .993 | .943 | .998 | .995 | .997 | .993 | .999 | .987 | .998 | .982 | .999 | .941 | .985 | .999 | .994 | .998 | .997 | .999 | .999 | 1.000 | - |
| Correlation coefficient per row j | .942 | .999 | .999 | .955 | .994 | .992 | .995 | .976 | .998 | .998 | .979 | .989 | .999 | .999 | .996 | .997 | .998 | .999 | .995 | - | .997 |

Table 11. Regression Results between Updated and Actual Coefficients per Column and per Row for Q.P.-method.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Regression coefficient | 1.067 | .929 | .968 | .972 | 1.041 | .995 | 1.002 | 1.030 | .986 | 1.090 | .990 | .950 | .984 | .984 | .987 | .972 | .969 | .635 | 1.009 | .993 | - |
| per column i | (.030) | (.064) | (.007) | (.017) | (.023) | (.025) | (.015) | (.035) | (.009) | (.035) | (.004) | (.050) | (.056) | (.014) | (.018) | (.019) | (.018) | (.038) | (.008) | (.002) | - |
| Regression coefficient | .913 | .962 | .999 | 1.022 | 1.110 | 1.008 | .934 | 1.032 | .994 | 1.005 | 1.035 | .985 | .970 | .995 | .918 | .963 | .835 | .995 | .935 | - | 1.078 |
| per row j | (.059) | (.006) | (.005) | (.038) | (.017) | (.021) | (.025) | (.036) | (.005) | (.007) | (.037) | (.025) | (.006) | (.007) | (.032) | (.015) | (.010) | (.007) | (.021) | - | (.018) |
| Correlation coefficient per column i | .911 | .929 | .999 | .997 | .993 | .986 | .997 | .986 | .999 | .988 | .999 | .955 | .952 | .997 | .994 | .995 | .996 | .999 | .999 | .999 | - |
| Correlation coefficient per row j | .927 | .999 | .999 | .970 | .997 | .992 | .987 | .972 | .999 | .999 | .980 | .987 | .999 | .999 | .985 | .997 | .998 | .999 | .993 | - | .997 |

Table 12. Regression Results between Differential Values of Updated Coefficients and Actual Coefficients per Column and per Row for RAS-method.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Regression Coefficient | .099 | .208 | .087 | .099 | .061 | .043 | .037 | .090 | .045 | .264 | .017 | .163 | .118 | .030 | .040 | .033 | .050 | .013 | .045 | .000 | - |
| per column i | (.015) | (.036) | (.007) | (.010) | (.011) | (.016) | (.005) | (.024) | (.009) | (.015) | (.003) | (.051) | (.017) | (.006) | (.017) | (.008) | (.010) | (.005) | (.005) | (.000) | - |
| Regression Coefficient | .177 | .084 | .026 | .138 | .244 | .072 | .057 | .107 | .022 | .032 | .124 | .052 | .023 | .002 | .033 | .041 | .014 | .046 | .059 | - | .109 |
| per row j | (.036) | (.003) | (.004) | (.033) | (.016) | (.013) | (.013) | (.024) | (.012) | (.008) | (.022) | (.021) | (.004) | (.004) | (.016) | (.011) | (.012) | (.005) | (.011) | - | (.015) |
| Correlation coefficient per column i | .801 | .727 | .927 | .880 | .654 | - | .827 | .522 | .628 | .961 | .725 | .371 | .763 | .644 | - | .445 | .701 | - | .877 | 1.000 | - |
| Correlation coefficient per row j | .464 | .990 | .807 | .441 | .957 | .624 | .623 | .523 | .188 | .586 | .365 | - | .674 | - | .313 | .579 | - | .869 | .745 | - | .529 |

**Table 13.** Regression Results between Differential Values of Updated Coefficients and Actual Coefficients per Column and per Row for Q.P.-method.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Regression Coefficient | .122 | .229 | .038 | .053 | .094 | .096 | .050 | .117 | .020 | .164 | .012 | .180 | .225 | .046 | .045 | .070 | .072 | .168 | .019 | .010 | - |
| per column i | (.019) | (.042) | (.005) | (.013) | (.014) | (.017) | (.009) | (.024) | (.009) | (.016) | (.004) | (.031) | (.024) | (.010) | (.015) | (.012) | (.007) | (.005) | (.007) | (.002) | - |
| Regression coefficient | .239 | .038 | .005 | .145 | .131 | .076 | .114 | .134 | .012 | .019 | .132 | .065 | .037 | .015 | .082 | .037 | .169 | .020 | .064 | - | .091 |
| per row j | (.033) | (.006) | (.004) | (.020) | (.007) | (.012) | (.014) | (.021) | (.004) | (.006) | (.024) | (.021) | (.003) | (.006) | (.032) | (.015) | (.006) | (.006) | (.018) | - | (.014) |
| Correlation coefficient per column i | .773 | .699 | .825 | .592 | .722 | .632 | .639 | .615 | - | .822 | - | .637 | .875 | .553 | .282 | .659 | .871 | .992 | - | .781 | - |
| Correlation coefficient per row j | .696 | .752 | - | .676 | .973 | .518 | .775 | .600 | .424 | .436 | .618 | .131 | .927 | - | - | - | .987 | .557 | .244 | - | .786 |

1) Figures between brackets represent standard errors of estimation.