SOME METHODS FOR UPDATING INPUT-OUTPUT TABLES

P. NIJKAMP and J.H.P. PAELINCK Netherlands Economic Institute, Rotterdam

1. Introduction.

One of the serious problems in regional analysis and forecasting is the lack of adequate and reliable data. Particularly, detailed interindustry models based on "technical coefficients", the so-called input-output models (I-O models), require a lot of information. It is altogether timeconsuming and costly to estimate a yearly I-O table. On the other hand, the technical coefficients once estimated are inherently unstable over a series of years. So, as is well known, the problem arises as to how to adjust a known I-O table on the basis of a limited quantity of information for later time periods.

One of the methods used to adjust a known I-O matrix from a certain basis year with the aid of known row and column totals from a later year is the socalled RAS-method. Expositions of this method are contained among others in

Stone (1963), Paelinck and Waelbroeck (1963), Bacharach (1965), Schneider (1965), Theil (1966), Lecomber (1971), Glattfelder and Váczi (1972), Mazys (1972) and Van Straelen (1972). The major part of these authors do not only discuss the properties of the RAS-method itself, but attempt to solve certain shortcomings of the RAS-method. In addition to the RAS-method some alternative methods of updating I-O tables are developed, viz. the statistical correction method developed by Tilanus (1965) and the linear programming method developed by Matuszewski et al. (1964). A comparison of the RAS-method with the statistical correction method is contained in Tilanus (1965), while a comparison of the RASmethod with a linear programming method can be found in Schneider (1965).

In this paper attention will be paid to an alternative way of adjusting technical coefficients, viz. *a quadratic programming* method. An analytical expression for the updating procedure will be derived, and next the quadratic programming approach will be tested for I-O data of the Belgian economy. The results will be compared with the RAS-results obtained for the same data of the Belgian economy by inspecting the standard errors of the projections. For that reason first a brief exposition of the RAS-method will be presented.

2. The RAS-method.

The RAS-method or biproportional method of updating I-O matrices attempts to gauge simultaneously two effects in the adjustment procedure, viz. (1) relative shifts in the required input proportions of a certain activity (i.e., substitution), and (2) changes in the productivity (i.e., less inputs per unit of output). Both effects are assumed to exert a systematic *uniform* influence upon the rows and columns of I-O tables. The substitution effect requires a systematic adaptation of the *rows* of an I-O table, while the productivity effect requires a systematic adaptation of the successive *columns* of an I-O table. A uniform adjustment of the rows is obtained by premultiplying the I-O matrix with a diagonal matrix r, while a uniform adjustment of the columns is obtained by postmultiplying the I-O matrix with a diagonal matrix s. Therefore, a new I-O matrix A^{*} is related to an I-O matrix A from a previous period as :

A* r A s

(2.1)

The previous adjustment is possible only if r and s are known. The estimation of r and s for a certain year is based on the row and column totals of the year concerned. The following row and column data are necessary : the vector of sectoral production levels (\underline{x}) , the vector of primary inputs per sector (\underline{v}) , and the vector of final demand per sector (\underline{f}) . By means of these data the total

intermediate output of commodities (\underline{u}) and the total intermediate input into commodities (\underline{y}) can be calculated, respectively, as :

$$\underline{\mathbf{u}} = \underline{\mathbf{x}} - \underline{\mathbf{f}}$$

and :

By making use of the balance equation for supply and demand in the classical I-O model, viz.,

$$\underline{x} = A^* \underline{x} + \underline{f}$$
(2.4)

it can easily be derived, that

$$\underline{u} = A^* \underline{x}$$
 (2.5)

Analogously, with the aid of the balance equation for production value and factor costs, viz.,

$$\underline{x} = x(A^{\star}) \cdot \underline{i} + \underline{v}, \qquad (2.6)$$

one can derive that :

$$\underline{y} = x(A^{*})^{*} \underline{1}$$
, (2.7)

where x is a diagonal matrix the diagonal elements of which are the elements of x, and where i is a unit (summation) vector.

substitution of (2.1) into (2.3) and (2.7) yields .

$$r A s x = u$$
 (2.8)

and

where <u>r</u> is a vector containing the diagonal elements of r. The systems (2.8) and (2.9) are a set of nonlinear equations containing the unknown elements of r and s. Since the number of equations is equal to the number of unknown elements, this system can, in principle, be solved.

The solution procedure itself is an iterative method converging towards the solution in a series of successive steps. The initial step is to insert into (2.8) the unit matrix as a preliminary solution for s and next to solve for the resulting value of r. Then, the latter value is substituted into (2.9) in order to determine a new value for s. Once this value has been calculated, one switches again to (2.8) in order to derive a new value for r and so forth, until the final solution is approximated up to a required degree of precision.

The convergence and uniqueness of this RAS-procedure are discussed by Bacharach (1965).

It is obvious that the RAS-method is based on some rigorous assumptions in particular the assumption of a *uniform* effect over each column and over each row. As a counter example, Paelinck and Waelbroeck (1963) observed a bad estimation of the substitution effect in the case of the coal industry, since coal was used as a raw material in the coke industry and as a fuel input elsewhere. By eliminating *a priori* these elements from the RAS-procedure and by making an independent estimation of these elements the RAS-procedure can be applied to the remaining elements, taking into account the prior information concerning the previous elements. In general, by means of prior information the quality of the adjustments is considerably enlarged (so-called "truncated" RAS-method).

3. Programming Methods for Updating I-O Tables.

As mentioned, another method of updating the technical coefficients of an I-O matrix was a linear programming method developed by Matuszewski et al. (1964). This method minimizes the relative deviations between the original value and the adjusted value of the coefficients of an I-O table. If the coef-

ficients of the original matrix A and of the updated matrix A^{*} are denoted by a_{ji} and a^{*}_{ji}, (j = 1, ..., I; i = 1, ..., I), respectively, the minimand is : min $\omega = \sum_{i,j} \left| \frac{a_{ji} - a_{ji}^{*}}{a_{ji}} \right|$ (3.1)

This objective function has to be minimized subject to the conditions (2.5) and (2.7); the result is essentially a linear programming model, which can be solved by means of standard techniques.

One of the major drawbacks of the linear programming method is the fact that the results may yield *negative* values for the updated coefficients. By introducing additional constraints, viz. lower limits of a_{ji}^{\star} , non-negativity can be preserved. Another drawback of the linear programming method is its implicit rigidity : the solutions of the linear programming model are always *cormer* solutions. This implies that frequently zero-values will be found for the adjusted coefficients, unless lower limits are imposed *a priori* (these lower limits are frequently rather arbitrary). An excessive positive variation of the coefficients can be prevented in a similar way by imposing arbitrary upper limits on the individual elements. Furthermore, corner solutions are often rather rigid with respect to minor changes in the minimand, so that often in the case of (3.1) a small shift in a_{ji} will exert no influence at all. It was shown by Schneider (1965) that linear programming methods tend to provide

adjustments with a lower quality than those of the RAS-method.

For that reason in this paper an alternative method of adjusting I-O coefficients will be derived, viz. a quadratic programming method. This method is less rigid than the linear programming method, and was first proposed by Friedlander (1961) for demographic projections. The purpose of this paper is to derive an *analytical* expression for the adjusted coefficients, and to compare the quality of the adjustments with those obtained by a RAS-method. In a next paragraph the quadratic programming method will be set out in more detail.

4. A Quadratic Programming Approach for the Adjustment of I-O Coefficients.

Instead of the objective function (3.1) it will be assumed here that the quadratic deviations between the original values and adjusted values of the I-O coefficients are to minimized. In order to prevent excessive variations in smaller coefficients the *relative* quadratic deviations are minimized. Therefore, the following quadratic programming (Q.P.) model arises (taking account of (2.5) and (2.7)) :

$$\begin{pmatrix} \min \omega = \frac{1}{2} \sum_{i,j} \left(\frac{a_{ji} - a_{ji}^{*}}{a_{ji}} \right)^{2} \\ \text{s.t.} \\ \frac{u}{2} = A^{*} \frac{x}{A} \\ \frac{y}{2} = x (A^{*})^{\circ} \frac{1}{2} \end{cases}$$
(4.1)

Next, one may define :

$$a_{ji}^{\star} = a_{ji} + \Delta_{ji}$$
 (4.2)

or :

$$A^* = A + \Delta$$
 (4.3)

Substitution of (4.2) and (4.3) into (4.1) yields :

$$\begin{pmatrix} \min \omega = \frac{1}{2} \sum_{i,j} \left(\frac{\Delta_{ji}}{a_{ji}} \right)^2 \\ \text{s.t.} \\ \frac{\mu}{2} = \left(A + \Delta \right) \times \\ \frac{\mu}{2} = x \left(A + \Delta \right)^* \underline{i} \\ \end{pmatrix}$$
(4.4)

By defining :

$$\frac{du}{du} = \frac{u}{Ax}$$

and :

$$\frac{dy}{dy} = \frac{y}{dy} = xA' \frac{1}{dy}$$
(4.6)

one obtains instead of (4.4) :

$$\begin{pmatrix} \min \omega * \frac{1}{2} \sum_{i,j} (\frac{\Delta_{ji}}{a_{ji}})^{2} \\ \text{s.t.} \\ \frac{du}{dy} * \Delta \cdot \underline{i} \\ \frac{dy}{dy} * \Delta \cdot \underline{i} \\ \end{pmatrix} (4.7)$$

The Lagrangean function L associated with (4.7) is :

$$L = \frac{1}{2} \sum_{i,j} \left(\frac{\Delta_{ji}}{a_{ji}} \right)^2 - \underline{\lambda}' \left(\Delta \underline{x} - \underline{du} \right) - \underline{\mu}' \left(\widehat{x} \Delta' \underline{i} - \underline{dy} \right), \qquad (4.8)$$

where $\underline{\lambda}$ and $\underline{\mu}$ are vectors of Lagrange multipliers associated with the previous systems of balance conditions. The Lagrangean function can be written in terms

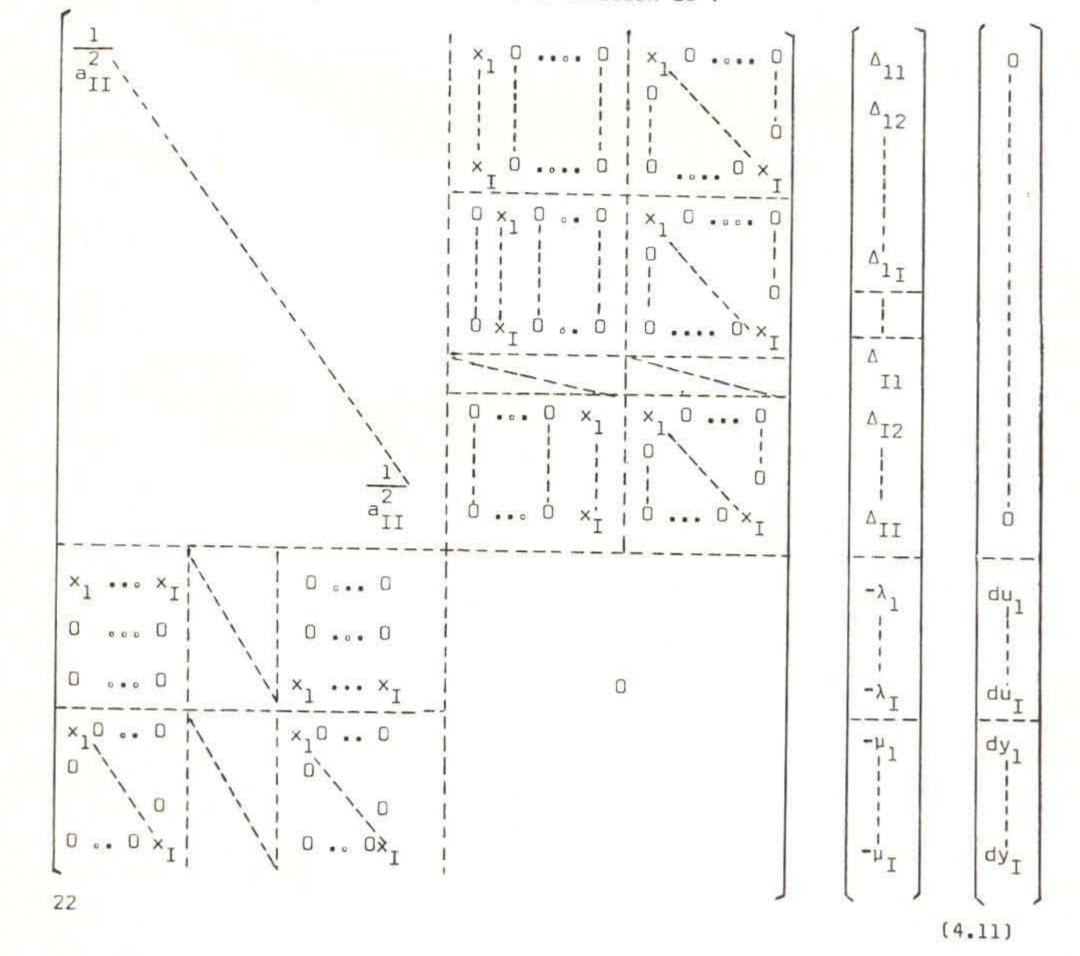
of individual elements as :

$$L = \frac{1}{2} \sum_{i,j} \left(\frac{\Delta_{ji}}{a_{ji}} \right)^{2} - \sum_{i,j} \lambda_{j} \Delta_{ji} x_{i} + \sum_{j} \lambda_{j} du_{j} - \sum_{i,j} \mu_{i} \Delta_{ji} x_{i} + \sum_{i} \mu_{i} dy_{i}$$
(4.9)

The first-order conditions associated with (4.9) are :

$$\begin{cases} \frac{\partial L}{\partial \Lambda_{ji}} * \frac{\Lambda_{ji}}{2} - \lambda_{j} x_{i} - \mu_{i} x_{i} = 0 \quad \forall i, \forall j \qquad (4.10) \\ \frac{\partial L}{\partial \lambda_{j}} * \sum_{i} \Lambda_{ji} x_{i} - du_{j} = 0 \qquad \forall j \\ \frac{\partial L}{\partial \lambda_{i}} * \sum_{j} \Lambda_{ji} x_{i} - dy_{j} = 0 \qquad \forall i \end{cases}$$

The second-order conditions for a minimum are obviously satisfied (a convex minimand defined on a convex set of side-conditions). The previous first-order conditions can be represented in matrix notation as :



The latter system contains $I^2 + 2 I$ equations with I^2 unknown variables Δ_{ji} , I unknown variables λ_j and I unknown variables μ_i . Such a linear system of equations can be solved in principle, if all equations are independent; in other words, if the matrix of coefficients is non-singular. There is, however, a strong dependency among the equations, associated the side-conditions, because the total change in intermediate production should be equal to the total change in intermediate requirements :

 $\sum_{i} du_{i} = \sum_{i} dy_{i}$ (4.12)

or :

 $\underline{i}^{\circ} \underline{du} = \underline{i}^{\circ} \underline{dy}$ (4.13)

This can formally be proved by substituting (4.5) and (4.6) into (4.13), viz.

 $\underline{i}^{\circ} \underline{u} = \underline{i}^{\circ} A \underline{x} = \underline{i}^{\circ} \underline{y} = \underline{i}^{\circ} \underline{x} A^{\circ} \underline{i}, \qquad (4.14)$

and next by substituting (2.5) and (2.7) into (4.14) :

$$i' A^* \times - i' A \times = i' \times (A^*)' i - i' \times A' i$$
 (4.15)

or :

(4.16)

<u>i' Δx * i' × Δ' i</u> * <u>i' Δx i</u> * <u>i' Δx</u> , q.e.d.

Therefore, one of the equations for the side-conditions can be dropped from the coefficient matrix of (4.11). This implies that one row and one corresponding column can be eliminated, so that the order of the coefficient matrix becomes $(I^2 + 2I - 1) \times (I^2 + 2I - 1)$. The resulting system of equations can now be written in a condensed form as :

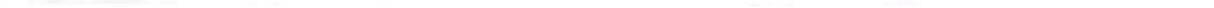
Bzsc

The unknown vector \underline{z} is of order ($I^2 + 2I - 1$); it contains I^2 unknown elements Δ_{ji} and 2I - 1 unknown elements λ_{j} and μ_{i} . In a similar way the elements of B and of \underline{c} can be considered. Assuming B is non-singular, one can easily solve \underline{z} as :

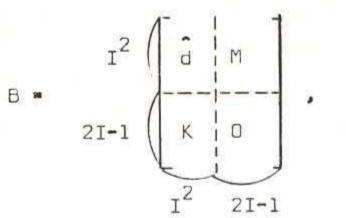
 $z = 8^{-1} c$ (4.18)

23

(4.17)



In order to reduce the computational work one may carry out a partition of the matrix B as follows :



(4.19)

where d is the left upperpart of the matrix B, viz. the $(I^2 \times I^2)$ diagonal matrix with main-diagonal elements $\frac{1}{2}$. K, M and the zero-matrix D are defined in a similar way. It is easily seen, that K = M', so that K can be replaced by M'. The inverse matrix of B can be written according to the partition of B as :

Obviously, the matrices N, P, Q and R have to be determined such that the following condition (multiplicative form of the inverse) is satisfied :

$$\begin{bmatrix} d & M \\ m^{*} & 0 \end{bmatrix} \begin{bmatrix} N & P \\ Q & R \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$
(4.21)

latter system can be written, successively, as :

 $d N + MQ = I$
(4.22)

 $d P + MR = 0$
(4.23)

 $M^{*}N (+0Q) = 0$
(4.24)

 $M^{*}P (+0R) = I$
(4.25)

Premultiplication of (4.22) with M'd⁻¹ gives :

$$M'N + M'd^{-1} MQ = M'd^{-1}$$
 (4.26)

Substitution of (4.24) into (4.26) gives :

24

The



$$M^{*}d^{-1} MQ = M^{*}d^{-1}$$

or:
 $Q = (M^{*}d^{-1} M)^{-1} M^{*}d^{-1}$
(4.28)

Substitution of (4.28) into (4.22) gives the following solution for N :

$$N = d^{-1} - d^{-1} M(M'd^{-1} M)^{-1} M'd^{-1}$$
(4.29)

By premultiplying (4.23) with $M'd^{-1}$ one obtains : M'P + M'd^{-1}MR = 0

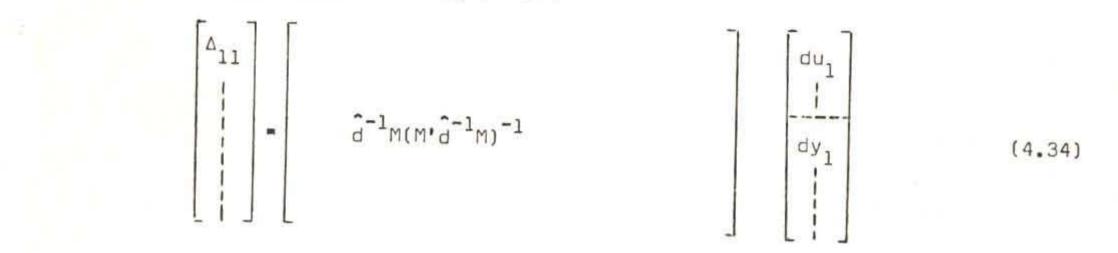
Next, by substituting (4.25) into (4.30) the solution for R is : $R = -(M^{\circ}d^{-1}M)^{-1}$ (4.31)

Finally, substitution of (4.31) into (4.23) leads to the following solution for P :

$$P = d^{-1} M(M'd^{-1}M)^{-1}$$
(4.32)

Therefore, the solution vector (4.18) can now be written as :

It can easily be derived from (4.33), that the solution vector for changes in the I-O coefficients is equal to :



25

(4.30)

The latter result implies a considerable reduction of computational requirements. The matrix d is a diagonal one, so that its inverse can be calculated directly. The inverse matrix of $M^{\circ}d^{-1}M$ can be calculated rather rapidly, since its order is (2I-1) x (2I-1). This means a considerable reduction compared to the inverse of the original matrix B of order (I^2 + 2I-1) x(I^2 + 2I-1). Therefore, the result can be calculated on almost each computer.

In the case of prior information concerning one or more of the elements Δ_{ji} the same procedure can be applied, viz. by eliminating these known elements a priori from the minimization procedure. Finally, it should be noted that the possibility of negative elements Δ_{ji} can be prevented by imposing the condition :

(4.35)

∆_{ji} ≥ -a_{ji} ,

albeit that in this case the previous efficient solution procedure cannot be applied, so that a more time-consuming algorithm for quadratic programming with inequalities has to be used. In the following paragraph the result obtained by means of the Q.P.-procedure will be compared with those obtained by means of a RAS-method.

5. A Comparison of the RAS-method and the Q.P.-method.

The two methods, discussed in par. 2 and par.4, will be compared on the

basis of I-O data for Belgium. An earlier analysis of changes in Belgian I-O matrices was carried out by Paelinck and Waelbroeck (1963). In this paragraph the same data will be handled and a comparison between the RAS-method and the Q.P. method will be made. In their article Paelinck and Waelbroeck presented an I-O matrix of 21 Belgian sectors, which was estimated both for the year 1959 (in constant prices). These I-O data are included in Table 1 and 2 of the Appendix, respectively.

The 1959 data for intermediate sectoral inputs (\underline{y}), for intermediate sectoral outputs (\underline{u}) and for sectoral production values (\underline{x}) are contained in Table 3 of the Appendix. On the basis of the 1953 I-O table and of the successive column and row totals from 1959 one may approximate the 'real' I-O matrix from 1959 by means of the 'updating-techniques' described previously.

The results, obtained by Paelinck and Waelbroeck by means of a RAS-procedure, are reprinted in Table 4 of the Appendix. Next, one may compare this 'updated' matrix for 1959 (i.e. Table 4) with the matrix actually estimated for 1959 (i.e. Table 2). In this way one may inspect the 'power' and accuracy of the RAS-technique.

In a similar way one may deal with the Q.P.-technique described in par.4. By applying the successive matrix operations a new set of 'updated' input-output-coefficients for 1959 was obtained. The results can be found in Table 5 of the Appendix. Our purpose is now (1) to analyse the predictive 'power' of this Q.P.-approach by comparing the updated coefficients with the actual coefficients, and (2) to compare the relative value of the Q.P.-technique and the RAS-technique mutually.

A first method of testing the accuracy of the updated coefficients is to calculate the relative mean deviation ('mean prediction error') between the updated and the actual coefficients in 1959, denoted by a_{ji}^{*} and a_{ji}^{0} , respectively. Such a mean deviation m can be defined as :

$$\Sigma |a_{ji}^{\circ} - a_{ji}^{*}|$$

$$\int a_{ji}^{\circ} a_{ji}^{\circ}$$

$$(5.1)$$

If the estimates a_{ji}^{\star} fall in the neighbourhood of the real coefficients a_{ji}^{0} , the mean prediction error becomes very small. This mean deviation can be calculated both for the RAS-technique and for the Q.P.-technique, and its value is a

measure for the relative reliability of the techniques used. The successive values of m for the RAS-technique and for the Q.P.-technique appeared to be equal to 0.094 and 0.105. This overall indicator shows that the mean prediction error of the Q.P.-technique is slightly higher than that of the RAS-technique. This relatively small difference suggests that there is no considerable difference between both methods.

A more accurate and detailed conclusion can be drawn by inspecting the sectoral mean deviations, both per column and per row. The mean deviation per column i is defined as :

$$m_{i} = \frac{\sum_{j=1}^{n} a_{ji}^{o} - a_{ji}^{*}}{\sum_{j=1}^{n} a_{ji}^{o}}$$
(5.2)

The latter measure is an indicator for the (in)accuracy of the I-O coefficients for the intermediate inputs into each sector i.

In a similar way one may define a mean deviation for each row j as :

 $m_{j} = \frac{\sum_{i=1}^{n} a_{ji}^{o} - a_{ji}^{*}}{\sum_{i=1}^{n} a_{ji}^{o}},$

(5.3)

which indicates the relative (in)accuracy of the updated coefficients for the intermediate outputs from each sector j.

The sectoral results of m_i and m_j both for the RAS- and for the Q.P.procedure are contained in Table 6 and 7 of the Appendix, respectively. These results confirm the previous provisional conclusion, that there is no considerable difference in the Q.P.- and the RAS-results. In these sectoral outcomes, too, there is a slight tendency for the mean prediction error of the Q.P.-method to be somewhat higher than that of the RAS-technique, viz. in 15 cases of the column deviations m (i = 1, ..., 21) and in 12 cases of the row deviations m j (j = 1, ..., 21).

In addition to a mean deviation one may inspect a relative quadratic deviation ('mean square prediction error') between the updated and the real coefficients. Such a relative quadratic deviation d can according to Theil (1966)

be defined as :

 $d = \frac{\sum (a_{ji}^{0} - a_{ji}^{*})^{2}}{i_{ji}}$

(5.4)

n

By taking the root of (5.4) one obtains the 'root-mean-square-prediction-error'. An alternative measure for the accuracy of the updating-technique is the socalled inequality coefficient q, defined as :

$$q = \frac{\sum_{j=1}^{\infty} (a_{ji}^{0} - a_{ji}^{*})^{2}}{\sum_{j=1}^{\infty} a_{ji}^{0^{2}}}$$
(5.5)

It is easily seen that $0 \leq q \leq 1$, when $0 \leq a_{ji}^* \leq 2a_{ji}^0$. Furthermore, it is obvious that $q \rightarrow 0$, when $a_{ji}^{*} \rightarrow a_{ji}^{0}$.

It is obvious that the latter measure bears some resemblance to the objective function of the Q.P.-procedure; this measure as well as the mean square predection error gives a higher (i.e., quadratic) 'penalty' to relatively higher deviations from the actual pattern. The values of q for the RAS-method and for the Q.P.-method are equal to 0.008 and 0.007, respectively. It appears that both values have a similar order of magnitude, albeit that now the Q.P.-method gives a slightly better result than the RAS-method. This confirms once more the



provisional conclusion that there is no significant difference in the quality of the Q.P.-method and the RAS-method.

It is evident that for each column and for each row separately also a relative quadratic deviation can be calculated. Analogously to (5.2) and (5.3) one may define :

$$q_{i} = \frac{\sum_{j=1}^{\sum_{j=1}^{\infty} a_{ji}^{2}}}{\sum_{j=1}^{\sum_{j=1}^{\infty} a_{ji}^{2}}}$$

(5.6)

and :

 $q_{j} = \frac{\sum_{i} (a_{ji}^{o} - a_{ji}^{*})^{2}}{\sum_{i} a_{ji}^{o}}$ (5.7)

The results of q_i and q_j both for the RAS-technique and for the Q.P.-technique are contained in Table 8 and 9 of the Appendix, respectively. These values show a global pattern nearly similar to that of m_i and m_j . There are only slight variations among the q_i 's and q_j 's, and there is no significant difference in the inequality coefficients neither for the columns nor for the rows. It appears that in 10 cases the relative quadratic deviation q_i (i = 1, ..., 21) of the

 $Q_{\circ}P_{\circ}$ -method is lower than that of the RAS-method, whereas the corresponding row indicator q (j = 1, ..., 21) is in 9 cases lower. Preliminarily, these results do not permit a firm conclusion in favour of one of both techniques.

An alternative way of obtaining more insight into the behaviour of the RAS-technique and of the Q.P.-technique is to carry out successively a regression analysis between the updated and the actual coefficients. By abandoning the intercept one obtains a regression line through the origin, the slope of which indicates whether the updating-technique concerned under- or overestimates the actual coefficients. The values of the regression coefficient for the RAS- and the Q.P.-technique appear to be equal to 0.954 and 0.962, respectively, whereas the successive standard errors of estimation are 0.008 and 0.004. The values of the regression coefficients are not suitable to discriminate between one of both techniques.

In addition, one can calculate the correlation coefficient associated with the previous regression procedure. The value of this correlation coefficient, denoted by r, indicates the degree to which there is a linear correlation

between the updated and the actual coefficients; in other words, the degree to which there exists a systematic scalter diagram between a_{ji}^{0} and a_{ji}^{\star} , which shows a close linear relationship through the origin between a_{ji}^{0} and a_{ji}^{\star} . The successive values of r for the RAS- and the Q.P.-technique appear to be 0.996 and 0.997. These results again lead to the conclusion that on the average the RAS- and the Q.P.-technique have about the same 'power' of updating or predicting I-O tables.

It is obvious that a similar procedure can be applied to each row and column separately. The regression coefficients as well as the correlation coefficients for all individual columns and rows can be defined in a similar way, viz. for each column i and for each row j. Their values are contained in Table 10 and 11 of the Appendix. Although there are some variations among the regression coefficients of the RAS- and of the Q.P.-technique, the results show globally a similar pattern. In both cases the regression coefficients are approximately equal to 1, so that both methods appear to provide good estimates of the actual pattern, although there are differences of a minor order. The successive corresponding correlation coefficients show in general a same order of magnitude, so that more and more the conclusion is justified, that both techniques possess the same quality in updating and predicting I-O coefficients.

Finally, one may inspect whether there is a systematic relationship between

the absolute difference in the actual and the updated coefficients (i.e., the prediction error) on the one hand, and the actual coefficients themselves on the other hand. So, by defining :

$$\Delta a_{ji} = |a_{ji}^{0} - a_{ji}^{*}|$$
 (5.8)

one may check whether there is a systematic linear relationship between ∆a ji and a⁰, both for the RAS-method and for the Q.P.-method. An (arbitrary) example of such a linear relationship is contained in figure 1.

The value of the regression coefficient between Δa_{ji} and a_{ji}^{0} indicates whether there is a systematic link between the absolute value of the prediction error per element and the value of the coefficient itself. These regression coefficients appeared to be equal to 0.071 and 0.058 for the RAS- and the Q.P.method, respectively, with respective standard errors 0.003 and 0.003. This result indicates once more that a definite conclusion in favour of one of both techniques is hard to draw, as appears also from the correlation coefficients, which are equal to 0.754 and 0.621, respectively.





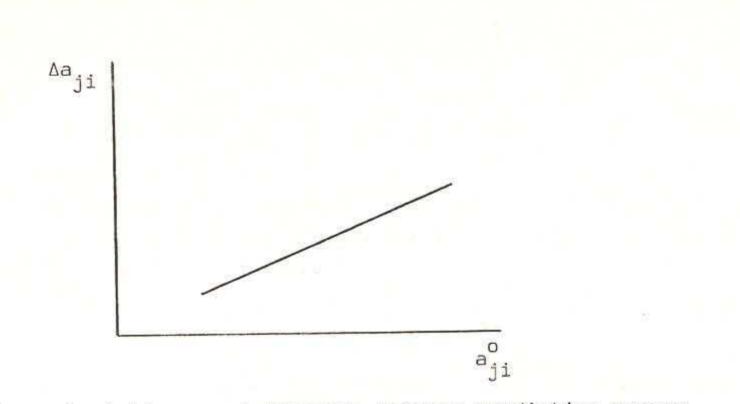


Figure 1. A linear relationship between prediction errors and actual values of I-O coefficients.

In a similar way one may calculate the regression coefficients and the correlation coefficients per row and per column both for the RAS- and the Q.P.method. These coefficients can be found in Table 12 and 13 of the Appendix, respectively.

The values of these coefficients appear to show considerable differences both per row and per column. There is, however, no systematic discriminating link either with respect to the individual sectors concerned or with respect to columns and rows. The only conclusion that can be drawn is that the size of the prediction error shows only a slightly positive relationship with respect to the level of the actual coefficient, both for the RAS- and for the Q.P.-technique.

By including an intercept into the previous regression procedure, the results were not essentially affected, except that the constant term itself appeared to be frequently non-significant.

6. Evaluation and Outline of Further Applications.

The results presented in par. 5 show that a unique conclusion either in favour of the RAS-technique or in favour of the Q.P.-technique cannot be drawn. The differences in the mean prediction errors, in the mean square errors and in the regression and correlation coefficients do not permit a definite conclusion. The specific property of the Q.P.-method is that it tends to truncate large deviations from an initial value of a coefficient owing to the quadratic 'penalty' function, as can be illustrated by inspecting the results for the elements a 23 and a43, both for the RAS- and for the Q.P.-technique. This suggests that the Q.P.-method might be helpful in the case of short-run adaptations of I-O tables, when considerable changes in I-O coefficients are less acceptable. Preliminarily, the general conclusion may be that the RAS-technique and the Q.P.-technique are







almost equivalent methods in updating problems.

Finally, it should be noted that the previous updating techniques can not only be applied in the case of I-O models, but also in demographic, traffic and modal-split models, and in many other allocation models which suffer from a lack of reliable and adequate permanent information.

A very specific application of the techniques described previously might be in pollution problems. As is well known, the relationship between the emission of pollution and the level of production can be represented by means of constant pollution-I-O-coefficients (P-I-O-coefficients). Such a relationship can be represented as :

where <u>e</u> represents a vector of order (K x 1) with elements e_k (k*1, - , k), representing the level of emission of pollutant k (carbon monoxide, sulfur dioxide, etc.). The relationship between the level e_k of pollutant k and the level x_i of production i (i = 1, ..., I) can be represented by means of the P-I-O-coefficient b_{ki} . The matrix B in (6.1), of order K x I, contains all these P-I-O-coefficients.

A very serious problem in pollution research is the estimation of the matrix B, as well as the yearly updating of this matrix. This matrix appears to be rather unstable, because the large-scale environmental deterioration forces entrepreneurs to implement alternative production processes leading to considerable changes both in the volume and in the 'mix' of emitted pollutants. It is extremely difficult to collect yearly data for these technical changes.

Therefore, an alternative approach might be to estimate the changes in B with the aid of known marginal data, given a known value of B in a certain basis year. Then the only problem is to collect data for \underline{e} and \underline{x} . In general, it will be possible to estimate the sectoral production levels \underline{x} , but the determination of the emission of pollution \underline{p} is frequently overloaded with difficulties.

In this case an alternative way may be to approximate the volume of emitted pollutants by means of data concerning the *concentration* of pollution at several observation points. In taking account of wind speed, wind direction and meteorological stability conditions the concentration of pollution at certain points can be 'transformed' into estimated emission values at the source by means of meteorological diffusion formulae. Such a diffusion analysis enables one to approximate the average emission of pollution at a certain region, given





a series of measurements on concentrations of pollution at several points (inside and/or outside the region). Once the unknown level of p has been estimated, one could use (6.1) as a side-condition in the Q.P.-technique, viz. by minimizing the relative quadratic differences between actual and past values of the P-I-O-coefficients, given the new marginal conditions (6.1).

The previous procedure shows still another possibility of the Q.P.technique. It serves to solve the number of degrees of freedom in updating problems without any restrictions on the number of side-conditions; for instance, in the previous P-I-O-case only horizontal additivity conditions were imposed, while the vertical conditions are disregarded. In this case the classical RAS-technique is not applicable, since it is based on horizontal and vertical marginal data, though a simplified RA- or AS-technique might give a first approximation. Whether Q.P.- or RAS-method is better in this case has still to be investigated.

References

BACHARACH, M., "Estimating Nonnegative Matrices from Marginal Data", International Economic Review, vol.6, nº 3, 1965, pp. 294-310.

- FRIEDLANDER, D., "A Technique for Estimating a Contingency Table, Given the Marginal Total and Some Supplementary Data", Journal of the Royal Statistical Society, series A, 1961, pp. 412-420.
- GLATTFELDER, P. and P. VACZI, "A Possible Extension of the RAS Method", Paper presented at the European Meeting of the Econometric Society, Budapest, 1972.
- LECOMBER, R., "A Critique of Methods of Adjusting, Updating and Projecting Matrices, together with some New Proposals", Discussion paper in Economics n° 40, University of Bristol, 1971.
- MATUSZEWSKI, T.I., R.R. PITTS and J.R. SAWYER, "Linear Programming Estimates of Changes in Input Coefficients", The Canadian Journal of Economics and Political Science, vol. 30, n°2, 1964, pp. 203-210.
- MAZYS, J., "The Methods of Estimation of an Input Coefficients Matrix", Paper presented at the European Meeting of the Econometric Society, Budapest, 1972。
- PAELINCK, J.H.P. and J. WAELBROECK, "Etude Empirique sur l'Evolution de Coefficients Input-Output", Economie Appliquée, Vol. 16, nº1, 1963, pp. 81-111.
- SCHNEIDER, H.M., "An Evaluation of two Alternative Methods for Updating Input-Output Tables", B.A. Thesis, Department of Economics, Harvard College, 1965.

STONE, J.R.S. (ed.), Input-Output Relationships, 1954-1966 (Series : A Programme for Growth), Chapman and Hall, London, 1963.

- THEIL, H., Applied Economic Forecasting, North-Holland Publishing Co., Amsterdam, 1966.
- TILANUS, C.B., Input-Output Experiments; The Netherlands 1948-1961, Rotterdam University Press, Rotterdam, 1965.
- VAN STRAELEN, "Forecasting Production Coefficients", Paper presented at the European Meeting of the Econometric Society, Budapest, 1972.







6.1	3 4	4	5	9	2	8	5	10 1	11 12	2 13	5 14	15	16	17	18	19	20
.001 .003 .002 .011	.002			1.006	025 .025		0. 000.	0. 900.	700. TOO	013.018	8 .010	100. 0	1.035	.008	.010	.005	ī
.676 .232 .003 .(C	. 024 . 0	.004 .032		.014 .0	.041 .0	.003 .010	770. 01		-005	010. 0	£00°	.004	.006	ī
002 -	I.		0	.051	1		.188 .0	.028 .0	.006 .001	01 .006	- 90	.001	.001	T	.001	.001	1
.015012		N	.036	36 .003	03 .01	N	-011 -0	.012 .0	.010 .013	13 .024	- +	.013	5.012	.006	.003	.006	ī
1		1	.026		1	•	. 035 .1	.174 .0	.012 .012	12 .006	00 . 002	2 .007	.002	1	.001	.002	ī
.004 .002 .011		1		001		0. 600.	. 002 . 0	. 008 .0	.026 .032	52.029	9.012		: 008	.038	.001	.036	I.
.001 .002 .001		0	.003	- 20		0. 900.	004 .0	.002 .0	.005 .004	04 .004	.005	5 .003	5 .020	.005	.002	.004	.018
.066 .044 .007		0	.048	18 .049		•	.073 .0	.020.0	.022 .021	21 .056	6 .030	0.018	.092	7:0.	.019	.010	ı
.026005		õ		1	0.	.006			199	006	039	1	1	.002	1	1	
009 .018	910.90	00	.035	. 52	•	.002 .0	.008	•	- 043 -		026	1	1	1	.001	.001	1
.011 .020 .052	20.052	N	100.	. 10	•	.044 .017		600.	022	22 .013	3.043	2 .007	.002	400.	.005	· 003	a,
.001 .014 .003	14 .003	3	0	.046 .030	700. 007		.004	0.	- 022 -	024	4 .062	2 .015	.036	.005	.015	•009	1
.006 .004 .006	00° +006	90	.013	- 21		.002 .0	028 .0	003 .0	.006 .004		143	3 .002	.001	.003	.010	.002	Ę
.002 .007 .041		ST	.005	100. 20	00.10		.002 .0	. 004 .0	.004 .005	700. 20	- 10	.006	100. 3	700.	•003	.002	113
007		50	.016	1	001		E	•	.006 .003		001	1	.001	.002	.002	.003	r
004		8	. 004	04 .002		.002 .002	•	002 .0	.002 .011	11 .009	500- 60	007	1	.034	600.	.008	1
002		8			ĩ	ĩ	1	ī	044	- 54	1	.013	1	1	.285	I	1
1	1	1	.022	- 22			1	T.	1		1	.004	ı.	.046	1	.120	ī
010 -			.012	2.016	6 .002		001	•	005 .001	100.10	N .003	5 .004	. 020	.006	.003	,	.006
E E	E E	1	•	5		1		Ē	E.	į	I.	Ð	I.	Ē	£	ï	ţ.
02	025	0.1		1	00.	01		1	039	5	.003	5.001	3	1	9	0	,

(2)

	sectors	-	2	r	4	5	9	2	8	O1	10	:	12	13	14	15	16	17	10	0)	50	5
0	oil	i	.005	.003	.011	.004	.016	.008	.028	. 01 :	C17	60.	600.	.030	.009	.003	.037	600.	.011	100.	,	1
2	coal	.001	9	.671	.192	100.	600.	.002	.013	.007	.023	.002	.005	.056	ı	.003	.004	.001	.002	400.	ı	1
N	coke and gas	1	.002	ł	.002	,	070.	ı	t	161	.025	.006	.001	.006	ı	.001	.001	Ē	.001	.001	1	1
4	electricity	500-	.024	.014	í.	.018	.036	.004	.018	.010	.015	.010	.013	.024	3	.015	.014	.003	.004	.008	1	x
5	various industries)	а	1	i	t	.023	1	ī	.033	.106	.011	.011	.006	.002	100.	.002	ĩ	.001	.002	1	Ē
0	chemistry	.036	.014	.005	.002	.015	ı	.001	.011	.002	600.	.031	.035	.033	.016	.031	.010	.052	.003	-057	3	3
4	finances	1	200.	.001	.002	.001	.co3.	1	100.	.004	.002	.005	.004	·004	.006	·003	.021	.005	.002	.004	.018	1
00	transport/ 7	-032	.017	.072	.044	.008	540.	.048	ī.	.083	.028	.023	.024	.055	.032	.019	.092	.018	.019	011	1	1
5	iron and steel		.018		1	.005	1	1	.005	а	ä	.205	a	.006	.039	1	ł	.002	ı	1	1	1
- E	non-ferrous metals	1	,		.008	.019	.030	I	.002	.007	1	.040	I	ĩ	.026	i.	ı	ï	.001	.001	1	1
	metal working	.010	.028	.012	.022	.063	700.	1	• 056	.018	.014	ī	.023	.014	740.	100.	.002	.004	.012	.004	3	1
12		700.	.019	.001	.014	.010	.044	.033	.007	.005	1	.022	ı	.027	.062	.017	.041	900.	.017	.011	Ļ	
n N	construction }	ı.	.001	.006	.004	.007	.011	ŝ	.002	.027	.003	.006	.004	1	.143	.002	.001	.003	.010	.002	a.	1
4 4	construction	.001	.008	.002	100.	.041	.004	.001	.005	.002	.004	.004	.005	.007	ł	900.	700.	100.	.003	.002	.118	1
	eather/textile	1	.002	1	ī	.008	.014	1	.001	ı	Ĩ	.006	.003	i	.001	ı	.001	.002	.002	500-	1	1
0	commerce	,I	001	1	1	.004	500.	.002	.002	.002	.002	.002	.010	600.	.004	100.	ł	.033	.009	.008	I	1
14 4	agriculture/	1	1	1	1	.002	ï	1	1	ı	ı	î	.016	ï	ľ	.013	Ę	t	.286	Ē	a,	1
14	Cood Chansett/Kinsato	ŧ.	č	I.	I.	t	.019	T	1	1	1	ī	ı	1	2	.004	a	.049	<u>a</u>	.114	1	1
-	various services	.002	.003	1	.010	2	.010	.016	.002	.001	t	.005	.001	.001	£00°	.004	.021	•006	.003	ï	.006	
- Ja	notel	1	ï	I.	ı	ı	ï	ţ	E	Ē	ŧ	1	1	I	•	1	ı.	1	ı	1	1	1
- Jan	government	1	1	1	ä	.027	9	ġ.	.001	а	ı	ï	.032	ī	.003	.004	J,	ï	I.	ī	ţ.	1

Table 3. Column and Row Totals from 1959 (10³ B.F.)

m

N

> = >

-	sectors	-	2	ы	4	5	9	2	ω	6	10	11	12
	1.5		100	.002	.004	.002	.011	100.	.028	.010	600.	.008	.007
- 0	110	001	1	.615	.175	.002	.014	£00°	.020	600.	.024	.002	900.
U 11	coat roke and gas		.002		.002	ä	.040	ï	r	.165	.022	.005	.001
1 5	rio!	005	.023	.024	t	.015	.038	.004	.013	.013	.012	.011	.012
t u	various industries		,	Î	6	1	.021	ł	4	.031	.135	.010	600.
1 4	chemistry.	041	.016	700.	.003	.015	1	.001	.011	.003	600.	.033	.034
7 (1	.006	.001	.002	.001	•003	1	.008	.004	.002	.005	.003
- a	transport/	.031	.014	.094	.052	.008	.044	.050	ï	.074	.018	.022	.018
0 0	communication	1	.019	.038	1	.005	i	1	.006	9	ï	.202	I
1 0	Sno	I	1	I	.010	.019	.031	1	.002	.008	I.	.041	1
		1007	.020	.019	.029	.068	.008	Ę	.052	.021	.010	а	.022
- 0	TONE	006	.029		.017	.009	.043	.032	100.	.004	i.	.023	t
4 1	construction	1	100-				.012	1	.002	.028	.003	.006	• 003
1 -	materials [001	.007			.041	.004	.001	.006	.002	.003	,004	.004
¢ ⊔	Lother/textile	1	002				.014	î	.001	I.	ı	.006	.002
n u	TCGUTCH/ VANA	1	.001	1	I.	.004	.003	.002	.002	.002	.002	.002	.008
2 1		I	•		1	.002	3	ï	I	1		1	.032
	forestry/fishery	Л	1	1	Ţ	Ĩ	.018	1	J.	1	3	ï	ı
0 0	various services	.002	.003	1	.01	ä	.011	.016	.002	.001	T.	· 005	.00
00		1	1	1	I	X	ŝ	•	640	1	а	1	1
10	øove mnent	1	1	Ē	E.	.029	-	1	001	ł	Ĩ	E	.036

		-	~	£	4	5	9	7	ω	6	10	::	12	13	14	15	16	17	18	19	50	51
						0	011	007	.028	010	600	. 008	. 100	021 .	. 011 .	. 600	. 040	600.	013 .	.006	a	204
-	oil	•	100			200			020		024	002	. 000.	.051		. 003	. 000.	.002	. 200.	.004	i	.440
01	coal	6				200				165	000			005	1		.001		001	.001	1	.246
103	coke and gas	1	.002	ı.	.002									000	1		014	.007	004	.008	t	.248
4	electricity	.005	.023	.024	1	.015		•004	.013											200	ı	225
LC	various industries	1	J.	ĩ	Ę.	1	.021	ł	1	• 031	.135	.010	. 600.	600			200.					202
		041	.016	700-	.003	.015	1	.001	.011	· 003	600.	.033	. 034 .	.039	.015 .	.032	.010	053	.002	200.		010.
D	Chemistry			0	000		N	1	.008	.004	.002	.005	. 003 .	004	. 000.	. 003 .	. 019 .	.005	.002	004	013	.096
-	finances /							050			018	.022	018	.059	.030	.019	. 693	.018	.022	011	9	.677
00	communication)	.031	-014	.094	200.		2		M						.040	4	1	.002	ł	1	I.	.319
a	iron and steel	1	610.	200.				G		000			9		.025	1	ı		.001	100	1	.138
0	non+ferrous metals	Ē.	1	L	.010			1	200	000			C	C	050	000	2003	.005	.007	.004	1	.353
-	metal working	100-	.020	.019	.029	.068		t	• 052	120.	010.								018	011	1	351
2	Vood/paper	.006	.029	.002	710.	600.	.043	.032	.007	.004	i.	620.	ı	070.	· · ·							220
4 1	construction	140	100	00B				I	.002	.028	£003	.006	.003	ĩ	.140	.002	.001	.003	.011	.002	1	.255
5	materials 🕈			200				100.	.006	.002	.003	,004	.004	100.	1	.006	.007	100.	.003	.002	.118	.234
-1	construction	3						I	100	, i	1	900.	.002	1	.001	ı	.001	.002	.002	.003	1	.041
5	leather/textile	r -	100			700			.002	.002	.002	.002	.008	600-	.004	100.	a	.034	.010	.008	1	.098
D:	commerce	1		1 9		200		•	I	1	Ľ	1	.032	1	1	.012	ı	ī	.238	ı	1	.329
-	forestry/fishery]	1	0				100			1	1	ī	1	ı	t	•004	h.	.044	ï	.118	1	.184
0	food	1 20	ECC	•	1 10	i 0			.002	.001	t	.005	.001	.001	.003	.004	.019	.006	.003	I.	900.	¥60°
01	AGTIQUE SELVICES		•	ŝ					in an	1	3	,	1	1	i	1	1	1	ä	3		I
20	hotel	1	1	1	t	1		•					920	19	2003	001	1	3	Ĩ	I	t	.070
2	government	1	I.	E	122	.029	1	1	-00-	ŧ	Ē	E	010.									



	sectors	-	C1	2	4	5	9	1	ထ	6	10	11	12	***
	011	I	.006	.001	.003	.002	-011	.006	.030	.019	.009	100.	100.	0.
10	ccal	100.	а	.647	.185	£00°.	.014	.004	.006	0	.029	.003	.003	0
30.)	coke and gas	r	.001	ĩ	.002	1	.039	1	1	.161	.027	.006	.001	0.
4	electricity	.004	.023	.017	ā.	.013	.041	.003	.014	.012	.012	.012	.013	0.
5	various industries	1	1	1	з	1	.022	1	1	031	.122	.010	.011	0.
10	chemistry	.040	.014	.004	.002	.012	1	.001	.010	.002	.008	.033	.029	0.
5-	finarces ,	1	700.	.8	.002	.001	£00°.	a	600.	.004	.002	500.	.004	0
CΩ	transport/ communication	.036	.015	160.	.055	100.	.040	.051	1	.077	.020	.021	.018	0.
51	Iron and steel	1	.019	.030	E	.005	E	I.	.006	Ľ	E	.203	ŧ	ō.
2	non-ferrous metals	ĩ	1	1	600.	.019	.031	3	.002	.003	3	.040	1	
***	metal working	.005	.118	.012	.023	.070	T00.	1	.062	.019	600.	I	.020	0
12	vooú/paper	500.	020.	100.	.015	.008	.041	• 031	1000.	· 004	ť.	.022	1	0
10	construction materials	Ľ	100.	.006	.004	.006	.013	1	.002	.029	£00°	.006	.004	
5	construction	.001	.008	.002	-00J	5:0-	.005	.001	900.	.002	+00·	.004	.005	0.
10	leather/textile	4	.002	1	ġ.	1000.	.015	3	.001	1	1	.006	.003	
9	commerce	I	.00	Ĕ	I.	.004	.004	.002	200.	.002	.002	.002	.010	0
5	forestry/fishery	1	1	1	a,	.002	1	T		t	ĕ	I.	.026	
0	Lood bool	r	ŧ	ī	ł	I	.019	ī	з	I	î	1	1	
0/	various services	.002	.003	4	.011	t	.011	.016	.002	.001	L	.005	.001	8
8	hotel	ł	a	1	1	ï	ł	I	21	1	1	T	1	
2	gove mment	ß	ï	n	ţ	020.	ţ	ī	100.	1	1	4	.035	

sectors	-	сч 1	ю	4	10	9	2	ထ	6	10	11	12	13	14	15	16	11		10	20	5
011	1	900.	.001	£00°	.002	.011	.006	.030	.019	.009	100.	100.	.020	.011	.003	570.	600.	.012	.005	1	- 31
CCAl	.001	а	.647	.185	£00°.	.014	.004	.006	10	.029	· 003	.003	070.	r.	.005	100.	.005	500.	.005	ï	1
coke and gas	r	.001	ĩ	.002	1	.039	1	į	.161	.027	.006	.001	.006	1	.001	.001	1	100	100.	1	I.
electricity	.004	.023	.017	ā.	.013	.041	.003	.014	.012	.012	.012	.013	.030	r	.016	.015	.007	.003	.007	ī	1
various industries	1	Ĩ	1	а	1	.022	2	9	031	.122	.010	.011	900.	.002	.007	.002	ł,	100	.002	i.	I.
chemistry	.040	.040 .014	.004	.002	.012	ĩ	.001	.010	.002	.008	.033	.029	.036	.013	.031	600.	.056	100.	.060	ä	'
finances ,	1	700°.	8	.002	100.	500.	T	600.	.004	.002	500.	.004	700.	.005	.003	018	.005	200.	.004	.019	1
communication	.036	·015	160.	.055	100.	.040	.051	١	.077	.020	.021	.018	010.	.028	.020	.089	.018	.023	010.	ì	I
Iron and steel	1	.019	.030	I.	.005	E	I.	.006	<u>r</u>	ł	.203	ŧ	.006	.040	I	ł	.002	э	1	ä	3
non-ferrous metals	ï	1	1	600.	.019	.031	3	.002	.003	9	.040	1	1	.025	I	Ľ	F	8	.001	ï	1
metal working	·002	.118	.012	.023	.070	100.	۱	.062	.019	600.	I	.020	.014	.055	.008	.002	.004	.006	.003	5	1
rood/paper	500.	.030	.001	.015	.008	.041	•031	100.	*00 *	Ű.	.022	ŧ.	.027	.064	-017	.039	500.	010	.010	1	ł
construction materials	I.	100.	.006	.004	.006	.013	ĩ	.002	.029	.003	.006	.004	я	.138	.002	:001	C00.	.012	.002	t	1
construction	.001	.008	.002	.007	.043	.005	.001	900.	.002	•004	.004	.005	100.	I	.006	.007	100.	£00.	.002	.012	3
leather/textile	1	.002	1	3	.007	.015	3	.001	1	1	.006	500.	а	.001	t	.001	2000.	.002	.003	1	1
commerce		.001	Ĕ	I	.004	·00	.002	.002	.002	.002	.002	.010	600.	.004	100.	1	.033	600.	.003	,	1
forestry/fishery	1	4	1	a.	.002	1	T	1	t	ĕ	Ľ.	.026	E	ĩ	.013	I	I	.284	1	1	Ĩ
L bocl	r	ŧ	ī	ł	1	.019	ĩ	з	ī	â	3	9	a	1	.004	1	970.	1	.116	t	1
various services	.002	.003	4	.011	t	.011	.016	.002	.001	ı	· 005	.001	.001	500.	•00 •	.019	.000	.003	ï	.006	1
hotel	ł	â	1	1	ï	ł	ī	3	1	1	T	1		1	æ	I.	4	I.	ī	ŗ	1
government	Į.	ï	n	ţ	020.	ţ	i	.001	1	ı	1	.035	1	£00.	.001	Ű	а	1	1	1	1

) The authors are indebted to Peter Mastenbroek, who carried out the computer analysis of the Q.P.-technique.

21	1	134		21	- 014	119		21	,	016		0 21	t	012
20	- 24 14		_	20		9		20	5	9		20		0
19	.042	.043		19	.063	960.		19	•003	.006		19	.001	.010
18	.047	.054		18	.174	.027		18	.001	· 003		18	.029	001
17	.049	.063		17	.082	.183		17	.004	.003		17	.005	000
16	.055	.041		16	.106	•092		16	£00°	.004		16	.003	500
15	.060	.047		15	.060	.209		15	.007	•000		15	.006	200
14	.041	.026		14	.071	•056		14	.002	1		14	•004	100
13	.126	.047		13	.176	.052		13	.020	.001		13	.063	~~~~
12	.204	.076		12	.209	060.		12	670.	.011		12	.051	
11	.031	.175		11	.039	.152		11	ĩ	.025		11	,	000
10	.238	.045		10	.210	•030		10	.074	.002		10	.032	1000
5	.075	.062		σ	.062	.026		σι	.004	•004		0	.002	
Ø	.119	.109		60	.163	.146		ω	.020	.023		ω	.025	
7	.043	.051	ro	7	.037	.121	rg.	7	.002	.007	100	5	\$CO.	
9	.067	080.	of Q.Pmethod	9	.105	960.	Coefficient of RAS-method and per Row	9	700.	.008	Q.Pmethod	9	.015	
10	.073	.183	of Q.P w.	10	.116	.118	t of RU w	5	.006	.065		5	.013	
4	.151	.150	n Error o per Row.	4	160.	.154	fficient per Row	4	.012	.041	Coefficient of and per Row	4	.006	
ы	0		Prediction Error Column and per Row	5	.055			10	.009	.001		10	.002	
2	33		Mean Pr per Col	2	.221	080	Inequality per Column	0	.069	700.	Inequality per Column	N	LSO.	
-	105		Table 7.		.160	152.	Table C.	1877	.014	.057	Table 9.	57	.022	
		4 _ 7			в	n 6	=	-		ц ^г		-	0	<u>ਜ</u>

21		65	.134		21	1	.119		21		.016		5		.012
20		ĩ	ř.		20	.014	r		20	1	1		20		1
01		.042	• 043		19	.063	960.		19	.003	.006		19	.00	.010
0	2. 1	.047	.054		18	.174	.027		18	.001	•003		00	.029	.00
17	-	•049	.063		17	.082	.183		17	.004	.003		17	.005	.029
34	2	.055	.041		16	.106	.092		16	£00°	.004		16	.003	.006
u	2	.060	.047		15	.060	.209		15	100.	.006		15	.006	.027
	+	.041	.026		14	.071	.056		14	.002	1		14	.004	.001
2.	2	.126	.047		13	.176	.052		13	.020	.001		13	.063	.002
	21	.204	.076		12	.209	060.		12	.079	.011		5	.051	.013
	-	.031	.175		11	.039	.152		E	ĩ	.025		11	1	.029
	10	.238	.045		10	.210	.030		10	•074	.002		10	.032	.001
	2	.075	.c62		6	.062	.026		σι	.004	.004		σ	.002	ï
1	ω	.119	.109		Ø	.163	.146		ω	.020	.023		Ø	.025	.027
	2	.043	.051	od	2	.037	.121	ođ	7	.002	.007	shod	4	, 00¢	.017
1000	9	.067	.080	of Q.Pmethod	9	.105	.096	Coefficient of RAS-method and per Row	9	700.	.008	Q.Pmethod	L.	.015	600*
	5	.073	.183		r0	.116	.118	ent of F Row	5	.006	•065	40	и 2	.013	.018
	4	.151	.150	on Erro	4	160.	.154	Coefficie and per 1	4	.012	.041	Coefficient		.006	.029
	£	.140	.036	Prediction Error Column and net Roy		.065	.032		15	.009	.001		MENTON	.002	i
	2	.193	.100	Mean	5	.221	080.	 E. Inequality per Column 	0	.069	700.	Thec		, os7	.002
	***	.105	.209	Table 7.	-	.160	153.	Table 8.		.014	.057	Table 9.		- 022	.073
-	-	n		H		в	n 1	=		g.	- .		=		,,,, ,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,

3 4 5 6 .140 .151 .073 .067 .140 .150 .183 .080 .0756 .150 .183 .080 .075 .150 .183 .080 .075 .150 .183 .080 .075 .150 .183 .080 .055 .9183 .080 .080 .055 .091 .116 .105 .055 .091 .118 .096 .032 .154 .118 .096 .032 .154 .118 .096 .032 .154 .118 .096 .032 .154 .118 .096 .031 .011 .065 .003 .001 .011 .065 .003 .001 .011 .065 .008 .001 .011 .065 .003 .001 .011 .065 .003 <tdd< th=""><th>a a a a a a a a a a a a a a a a a a a</th><th></th><th>7 .043 .051 .051 .037 .037 .002 .002 .002 .007 .007</th><th>7 8 9 1 .045 .119 .075 . .051 .119 .075 . .051 .109 .075 . .051 .109 .062 . .037 .165 .062 . .037 .165 .062 . .037 .165 .062 . .121 .146 .056 . .121 .146 .056 . .001 .025 .004 . .004 .025 .004 . .004 .025 .002 . .004 .025 .002 . .004 .025 .002 . .004 .025 . . .017 .025 . . .017 .025 . .</th><th>7 8 9 1 .045 .119 .075 . .051 .109 .075 . .051 .109 .075 . .051 .109 .062 . .037 .165 .062 . .037 .165 .062 . .121 .146 .026 . .121 .146 .026 . .002 .020 .004 . .001 .025 .004 . .004 .025 .004 . .004 .025 .002 . .004 .025 .002 . .004 .025 .002 . .004 .025 . . .017 .025 . .</th><th>Column and per now.</th><th></th><th>1</th><th>.183</th><th>r of Q. ow.</th><th>10</th><th>.116</th><th>.118</th><th>int of] low</th><th>5</th><th>.006</th><th>.065</th><th>ent of low</th><th>5</th><th>.013</th><th>.018</th></tdd<>	a a a a a a a a a a a a a a a a a a a		7 .043 .051 .051 .037 .037 .002 .002 .002 .007 .007	7 8 9 1 .045 .119 .075 . .051 .119 .075 . .051 .109 .075 . .051 .109 .062 . .037 .165 .062 . .037 .165 .062 . .037 .165 .062 . .121 .146 .056 . .121 .146 .056 . .001 .025 .004 . .004 .025 .004 . .004 .025 .002 . .004 .025 .002 . .004 .025 .002 . .004 .025 . . .017 .025 . . .017 .025 . .	7 8 9 1 .045 .119 .075 . .051 .109 .075 . .051 .109 .075 . .051 .109 .062 . .037 .165 .062 . .037 .165 .062 . .121 .146 .026 . .121 .146 .026 . .002 .020 .004 . .001 .025 .004 . .004 .025 .004 . .004 .025 .002 . .004 .025 .002 . .004 .025 .002 . .004 .025 . . .017 .025 . .	Column and per now.		1	.183	r of Q. ow.	10	.116	.118	int of] low	5	.006	.065	ent of low	5	.013	.018
6 .067 .067 .060 .105 .105 .105 .105 .006 .006 .007 .007 .007 .007 .007 .015 .009	a a a a a a a a a a a a a a	a a a a a a a a a a a a a a	7 8 .045 .119 .051 .109 .051 .109 .037 .165 .121 .165 .037 .165 .037 .165 .037 .165 .037 .165 .037 .165 .037 .126 .121 .146 .121 .146 .121 .146 .022 .020 .002 .020 .001 .025 .004 .025 .017 .021	7 8 9 1 .045 .119 .075 . .051 .109 .075 . .051 .109 .075 . .051 .109 .062 . .037 .165 .062 . .037 .165 .062 . .121 .146 .056 . .121 .146 .056 . .022 .020 .004 . .001 .025 .004 . .004 .025 .004 . .004 .025 .002 . .004 .025 .002 . .004 .025 .002 . .017 .025 . . .017 .025 . .	7 8 9 10 .043 .119 .075 .238 .051 .109 .075 .238 .051 .109 .075 .238 .051 .109 .052 .045 .037 .163 .045 .045 .037 .163 .045 .045 .037 .163 .046 .046 .121 .146 .026 .030 .121 .146 .026 .030 .121 .146 .026 .030 .001 .025 .004 .002 .004 .025 .004 .032 .004 .025 .032 .032 .004 .025 .032 .032 .011 .027 .032 .032		-	N			1	1		0	1	1	1	0		1	193
	a a a a a a a a a a a a a a	a a a a a a a a a a a a a a	7 8 .045 .119 .051 .109 .051 .109 .037 .165 .121 .165 .037 .165 .037 .165 .037 .165 .037 .165 .037 .165 .037 .126 .121 .146 .121 .146 .121 .146 .022 .020 .002 .020 .001 .025 .004 .025 .017 .021	7 8 9 1 .045 .119 .075 . .051 .109 .075 . .051 .109 .075 . .051 .109 .062 . .037 .165 .062 . .037 .165 .062 . .037 .165 .062 . .037 .165 .062 . .121 .146 .026 . .001 .023 .004 . .004 .025 .004 . .004 .025 .002 . .004 .025 .002 . .004 .025 .002 . .017 .025 .002 . .017 .025 . .	7 8 9 10 .043 .119 .075 .238 .051 .109 .075 .238 .051 .109 .075 .238 .051 .109 .052 .045 .037 .163 .045 .045 .037 .163 .045 .045 .037 .163 .046 .046 .121 .146 .026 .030 .121 .146 .026 .030 .121 .146 .026 .030 .001 .025 .004 .002 .004 .025 .004 .032 .004 .025 .032 .032 .004 .025 .032 .032 .011 .027 .032 .032		10	.067		Pmetho	9	.105	960*	AS-me the	9	.007	.008	l.Pmet	9	.015	600.
9 10 .075 .238 .062 .238 .062 .045 .062 .210 9 10 9 10 9 .074 .004 .074 .004 .074 .004 .074 .004 .074 .004 .074 .004 .074 .004 .074 .004 .074 .002 .030 9 10 9 .002 .002 .032 .002 .032	10 11 .238 .031 .238 .031 .045 .175 .045 .175 .050 .039 .210 .039 .210 .039 .210 .039 .210 .039 .030 .152 .030 .152 .031 .039 .032 .152 .033 .152 .034 .11 10 11 10 .025 .032 .025 .032 .025	11 11 28 .031 28 .031 10 .039 11 74 - 74 - 74 - 02 .025 02 .025 01 .029	31 35 52 25 225 225	12 .204 .076 .079 .079 .011 .011 .011			13	.126	.047		13	.176	.052		13	.020	.001		13	.063	.002
9 10 11 12 1 .075 .238 .031 .204 .062 .045 .175 .076 .062 .045 .175 .076 9 10 11 12 9 10 11 12 9 10 11 12 9 10 11 12 9 10 11 12 9 10 11 12 9 10 11 12 9 10 11 12 9 10 11 12 9 10 11 12 9 10 11 12 9 10 11 12 9 10 11 12 9 10 11 12 9 10 11 12 9 10 11 12 10 <	10 11 12 1 .238 .031 .204 .045 .175 .204 .045 .175 .076 .040 .175 .076 .210 .039 .209 .030 .152 .090 .074 - .079 .074 - .079 .074 - .079 .074 - .079 .074 - .079 .074 - .079 .074 - .079 .074 - .079 .074 - .079 .074 - .079 .075 .071 .071 .0725 .071 .071 .0732 - .079 .073 .073 .011 .073 .073 .011 .073 .025 .011 .073 .023 .013 <tr tr=""></tr>	11 12 1 11 12 1 11 11 12 11 12 .076 11 12 12 .011 13 .051 14 .051 15 .013	12 12 31 .204 75 .076 39 .209 52 .090 52 .079 25 .079 25 .079 25 .079 25 .079 25 .079 25 .079 25 .079 25 .079 25 .079 25 .079 25 .079 25 .079 25 .079 25 .079 25 .079 25 .079 25 .079		13 -126 -047 -047 -047 -047 -047 -047 -052 -052 -001 -001 -005 -005		14	.041	.026		14	.071	• 056		14	.002	1		14	·004	001
9 10 11 12 13 1 .075 .238 .031 .204 .126 .052 .045 .175 .076 .047 .052 .045 .175 .076 .047 9 10 11 12 13 9 10 11 12 13 .052 .210 .039 .209 .176 .052 .210 .039 .209 .176 .052 .011 12 13 1 9 10 11 12 13 .004 .074 - .079 .052 .004 .074 - .079 .052 .004 .002 .011 .001 .001 .004 .011 12 13 .011 9 10 11 12 13 .063 .005 .032 .033 .051 .063 .063	10 11 12 13 1 .238 .031 .204 .126 . .245 .175 .076 .047 . .045 .175 .076 .047 . .050 .175 .076 .047 . .050 .152 .076 .047 . .210 .039 .209 .176 . .074 .039 .209 .176 . .074 .039 .052 .052 . . .074 - .079 .052 074 - .079 .079 .052 074 - .079 .079 .001 074 - .079 .079 074 - .079 .071 	11 12 13 1 58 .031 .204 .126 . 58 .031 .204 .126 . 10 .039 .204 .126 . 11 12 13 . . 10 .039 .209 .176 . 74 - .079 .052 . 74 - .079 .052 . 74 - .079 .052 . 74 - .079 .052 . 71 12 13 . . 74 - .079 .052 . 71 12 .050 .052 . 71 12 .011 . . 75 - .051 . . 75 .011 .051 . . 75 - .011 . .	12 13 1 31 .204 .126 75 .076 .047 39 .076 .047 52 .076 .047 52 .090 .052 12 13 12 13 12 13 12 13 12 .050 .079 .052 .079 .052 .079 .052 .079 .052 .079 .052 .079 .052 .079 .052 .079 .052 .079 .052 .079 .052 .079 .052 .079 .052 .079 .052 .079 .052 .079 .052 .071 .001 .051 .063 .051 .063	13 13 126 .126 .047 .052 .052 .052 .055 .055 .005	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		5	.060	.047		15	.060	.209		15	.007	.006		15	.006	.027
9 10 11 12 13 14 .075 .238 .031 .204 .126 .041 .075 .238 .031 .204 .126 .041 .052 .045 .1175 .076 .047 .026 .052 .045 .175 .076 .047 .026 9 10 11 12 14 .071 .052 .210 .039 .209 .176 .071 .052 .030 .152 .090 .052 .056 .054 .074 .209 .075 .056 .076 .054 .074 .209 .052 .056 .056 .004 .074 .073 .079 .070 .056 .004 .002 .026 .011 .001 - .004 .002 .026 .011 .001 - .004 .001 .11 .071 .0	10 11 12 13 14 .238 .031 .204 .126 .041 .045 .175 .076 .047 .026 .045 .175 .076 .047 .026 .050 .175 .076 .047 .026 .030 .152 .090 .052 .071 .210 .039 .209 .176 .071 .210 .039 .209 .176 .071 .030 .152 .090 .052 .056 .031 .172 .090 .052 .071 .074 - .079 .076 .071 .074 - .079 .052 .056 .074 - .079 .072 .071 .001 11 12 13 14 .010 .011 .001 .001 - .032 .025 .011 .001 .004	11 12 13 14 58 .031 .204 .126 .041 59 .031 .204 .126 .041 51 .175 .076 .047 .026 10 .039 .204 .176 .071 10 .039 .209 .176 .071 74 - .079 .052 .056 74 - .079 .052 .056 01 .12 .13 .14 11 .12 .13 .14 11 .12 .13 .071 02 .025 .071 .076 .076 03 .152 .071 .071 .071 04 - .071 .076 .076 .076 05 .025 .011 .001 .002 .004 .001	12 13 14 31 .204 .126 .041 75 .076 .047 .026 75 .076 .047 .026 75 .076 .047 .026 75 .076 .047 .026 75 .076 .047 .026 75 .090 .052 .071 75 .090 .052 .071 75 .090 .052 .071 75 .079 .072 .071 75 .079 .052 .056 75 .071 .001 - 75 .011 .001 - 76 .071 .001 - 76 .073 .002 .004 76 .073 .001 .001	13 14 .126 .041 .126 .041 .047 .026 .052 .071 13 14	14 14 14 17 .026 14 14 14 14 14 14 14 14 14 14 14 63 .002 63 .004		16	.055	.041		16	.106	.092		16	•003	.004		16	•003	.006
9 10 11 12 13 14 15 .075 .238 .031 .204 .060 .041 .060 .062 .045 .175 .076 .041 .060 .047 .062 .045 .175 .076 .047 .026 .047 9 10 11 12 13 14 15 9 10 11 12 13 14 15 9 10 11 12 13 14 15 9 10 11 12 13 14 15 9 10 11 12 13 14 15 9 101 12 13 14 15 106 1004 .002 .001 .001 .001 .005 .005 101 9 10 11 12 13 14 15 105 9 10 </td <td>10 11 12 13 14 15 15 .238 .031 .204 .126 .041 .060 .238 .031 .204 .126 .041 .060 .045 .175 .076 .047 .060 .047 .045 .175 .076 .047 .060 .047 .05 .175 .076 .047 .060 .047 .10 11 12 13 14 15 .090 .050 .152 .090 .052 .076 .209 .010 .060 .074 - .079 .076 .070 .076 .006 .010 11 12 13 14 15 .016 .001 .11 .001 .001 .001 .006 .001 .010 .011 .001 .011 .001 .001 .001 .011 .011 .011 .012</td> <td>11 12 13 14 15 15 58 .031 .204 .126 .041 .060 55 .175 .076 .047 .060 .047 11 12 .076 .047 .060 .047 11 12 13 14 15 .04 10 .039 .209 .076 .047 .060 30 .152 .090 .052 .056 .209 30 .152 .090 .052 .056 .209 11 12 13 14 15 .07 11 12 13 14 15 .066 11 12 13 14 15 .066 11 12 13 14 15 .066 11 12 13 14 15 .066 11 12 13 14 15 .066 11</td> <td>12 13 14 15 1 31 .204 .126 .041 .060 75 .076 .047 .060 .047 75 .076 .047 .060 .047 75 .076 .047 .060 .047 75 .076 .047 .026 .047 75 13 14 15 .047 75 .079 .052 .071 .060 75 13 14 15 .071 75 13 14 15 .071 75 .079 .020 .000 .000 75 13 14 15 .001 75 .011 .001 .001 .001 .001 75 14 15 .001 .001 .005 75 15 14 15 .001 .005 .005 75 .015 .002 .004</td> <td>13 14 15 .126 .041 .060 .047 .026 .047 .071 .060 .176 .071 .060 .176 .071 .060 .176 .071 .060 .176 .071 .060 .052 .056 .209 .051 .002 .001 .001 .006 .001 .001 .004 .006 .002 .004 .006 .003 .004 .006 .005 .004 .006</td> <td>14 15 1 26 .041 .060 .047 27 .026 .047 .060 76 .071 .060 .047 76 .071 .060 .047 76 .071 .060 .047 76 .071 .060 .060 76 .071 .060 .071 20 .056 .209 .007 21 14 15 .006 14 15 .006 .001 21 - .006 .001 22 .002 .001 .006 14 15 .006 .001 14 15 .006 .006 63 .004 .006 .006 201 .001 .021 .021</td> <td></td> <td>17</td> <td>.049</td> <td>.063</td> <td></td> <td>17</td> <td>.082</td> <td>.183</td> <td></td> <td>17</td> <td>.004</td> <td>.003</td> <td></td> <td>17</td> <td>.005</td> <td>.029</td>	10 11 12 13 14 15 15 .238 .031 .204 .126 .041 .060 .238 .031 .204 .126 .041 .060 .045 .175 .076 .047 .060 .047 .045 .175 .076 .047 .060 .047 .05 .175 .076 .047 .060 .047 .10 11 12 13 14 15 .090 .050 .152 .090 .052 .076 .209 .010 .060 .074 - .079 .076 .070 .076 .006 .010 11 12 13 14 15 .016 .001 .11 .001 .001 .001 .006 .001 .010 .011 .001 .011 .001 .001 .001 .011 .011 .011 .012	11 12 13 14 15 15 58 .031 .204 .126 .041 .060 55 .175 .076 .047 .060 .047 11 12 .076 .047 .060 .047 11 12 13 14 15 .04 10 .039 .209 .076 .047 .060 30 .152 .090 .052 .056 .209 30 .152 .090 .052 .056 .209 11 12 13 14 15 .07 11 12 13 14 15 .066 11 12 13 14 15 .066 11 12 13 14 15 .066 11 12 13 14 15 .066 11 12 13 14 15 .066 11	12 13 14 15 1 31 .204 .126 .041 .060 75 .076 .047 .060 .047 75 .076 .047 .060 .047 75 .076 .047 .060 .047 75 .076 .047 .026 .047 75 13 14 15 .047 75 .079 .052 .071 .060 75 13 14 15 .071 75 13 14 15 .071 75 .079 .020 .000 .000 75 13 14 15 .001 75 .011 .001 .001 .001 .001 75 14 15 .001 .001 .005 75 15 14 15 .001 .005 .005 75 .015 .002 .004	13 14 15 .126 .041 .060 .047 .026 .047 .071 .060 .176 .071 .060 .176 .071 .060 .176 .071 .060 .176 .071 .060 .052 .056 .209 .051 .002 .001 .001 .006 .001 .001 .004 .006 .002 .004 .006 .003 .004 .006 .005 .004 .006	14 15 1 26 .041 .060 .047 27 .026 .047 .060 76 .071 .060 .047 76 .071 .060 .047 76 .071 .060 .047 76 .071 .060 .060 76 .071 .060 .071 20 .056 .209 .007 21 14 15 .006 14 15 .006 .001 21 - .006 .001 22 .002 .001 .006 14 15 .006 .001 14 15 .006 .006 63 .004 .006 .006 201 .001 .021 .021		17	.049	.063		17	.082	.183		17	.004	.003		17	.005	.029
9 10 11 12 13 14 15 16 1 .075 .238 .031 .204 .126 .041 .060 .055 . .062 .045 .175 .076 .047 .067 .041 .041 .052 .045 .175 .076 .047 .060 .045 .041 .052 .210 .039 .209 .176 .071 .060 .106 .052 .070 .079 .076 .071 .050 .092 .052 .070 .079 .076 .071 .050 .092 .054 .071 12 13 14 15 16 .004 .001 .001 .001 .001 .001 .003 .004 .001 .001 .001 .001 .001 .003 .004 .001 .011 .001 .001 .001 .001	10 11 12 13 14 15 16 1 .238 .031 .204 .126 .041 .060 .055 . .238 .031 .204 .126 .041 .060 .055 . .045 .175 .076 .047 .026 .047 .041 . .050 .152 .039 .176 .071 .060 .041 . .050 .152 .090 .052 .056 .209 .092 .074 .073 .076 .071 .060 .066 .066 .074 .079 .079 .076 .071 .060 .066 .074 - .071 .071 .07 .092 .092 .074 - .071 .071 .076 .006 .006 .071 .071 .001 .071 .001 .003 .004 .004 .001 .0	11 12 13 14 15 16 16 13 .031 .204 .126 .041 .060 .055 175 .076 .047 .026 .047 .041 11 12 .076 .047 .056 .041 .041 11 12 13 14 15 16 1 10 .039 .209 .176 .011 .060 .052 11 12 13 14 15 16 1 11 12 .050 .056 .209 .092 .092 11 12 13 14 15 16 1 14 12 14 15 16 1 1 14 12 14 15 16 1 1 14 12 14 15 16 1 1 11 12 101 .001 -015 </td <td>12 13 14 15 16 1 71 .204 .126 .041 .060 .055 75 .076 .047 .050 .055 .041 75 .076 .047 .041 .041 .041 75 .076 .047 .041 .041 .041 70 .071 .026 .047 .041 .041 70 .071 .026 .047 .041 .041 70 .072 .071 .060 .106 .042 71 .12 13 14 15 16 71 .071 .052 .056 .092 .092 71 .12 13 14 15 16 71 .001 .001 -006 .004 72 .011 .001 .001 .003 74 15 14 15 16 <tr td=""> 74 12</tr></td> <td>13 14 15 16 1 .126 .041 .060 .055 . .047 .026 .047 .051 . .047 .026 .047 .041 . .047 .026 .047 .041 . .041 .026 .047 .041 . .176 .071 .052 .047 .041 .176 .071 .050 .047 .041 .176 .071 .050 .056 .092 .176 .071 .050 .092 . .052 .056 .209 .092 . .051 .14 15 16 . .001 .0 .005 .004 . . .13 .14 15 16 . . .13 .14 15 13 .14 .15 . .</td> <td>14 15 16 1 26 .041 .060 .055 . 27 .026 .041 .041 . 26 .041 .060 .055 . 27 .026 .047 .041 . 26 .071 .060 .041 . 27 .071 .060 .041 . 28 .071 .060 .041 . 29 .071 .060 .062 . 20 .071 .060 .092 . 21 15 16 . . 20 .002 .001 .003 . . 21 15 16 . . . 21 15 21 21 </td> <td></td> <td>18</td> <td>.047</td> <td>.054</td> <td></td> <td>18</td> <td>.174</td> <td>.027</td> <td></td> <td>18</td> <td>.001</td> <td>.003</td> <td></td> <td>18</td> <td>.029</td> <td>001</td>	12 13 14 15 16 1 71 .204 .126 .041 .060 .055 75 .076 .047 .050 .055 .041 75 .076 .047 .041 .041 .041 75 .076 .047 .041 .041 .041 70 .071 .026 .047 .041 .041 70 .071 .026 .047 .041 .041 70 .072 .071 .060 .106 .042 71 .12 13 14 15 16 71 .071 .052 .056 .092 .092 71 .12 13 14 15 16 71 .001 .001 -006 .004 72 .011 .001 .001 .003 74 15 14 15 16 <tr td=""> 74 12</tr>	13 14 15 16 1 .126 .041 .060 .055 . .047 .026 .047 .051 . .047 .026 .047 .041 . .047 .026 .047 .041 . .041 .026 .047 .041 . .176 .071 .052 .047 .041 .176 .071 .050 .047 .041 .176 .071 .050 .056 .092 .176 .071 .050 .092 . .052 .056 .209 .092 . .051 .14 15 16 . .001 .0 .005 .004 . . .13 .14 15 16 . . .13 .14 15 13 .14 .15 . .	14 15 16 1 26 .041 .060 .055 . 27 .026 .041 .041 . 26 .041 .060 .055 . 27 .026 .047 .041 . 26 .071 .060 .041 . 27 .071 .060 .041 . 28 .071 .060 .041 . 29 .071 .060 .062 . 20 .071 .060 .092 . 21 15 16 . . 20 .002 .001 .003 . . 21 15 16 . . . 21 15 21 21 		18	.047	.054		18	.174	.027		18	.001	.003		18	.029	001
9 10 11 12 13 14 15 16 17 1 .075 .238 .031 .204 .126 .041 .060 .055 .049 .062 .045 .175 .076 .047 .060 .055 .049 .062 .040 11 12 13 .04 .063 .041 .063 .062 .210 .039 .209 .176 .071 .063 .063 .056 .030 .152 .090 .052 .056 .209 .183 .056 .030 .152 .090 .052 .056 .183 .056 .030 .152 .090 .056 .209 .183 .056 .051 .050 .205 .209 .205 .183 .056 .051 .050 .056 .209 .203 .203 .056 .051 .050 .050 .205	10 11 12 13 14 15 16 17 1 .238 .031 .204 .126 .041 .060 .055 .049 . .238 .031 .204 .126 .041 .060 .055 .049 . .045 .175 .076 .047 .026 .047 .063 .063 . .050 .175 .076 .071 .026 .092 .193 . . .063 .	11 12 13 14 15 16 17 58 .031 .204 .126 .041 .060 .055 .049 45 .175 .076 .047 .026 .041 .063 .049 45 .175 .076 .047 .026 .047 .063 .049 40 .039 .209 .176 .071 .060 .165 .17 10 .039 .209 .176 .071 .060 .165 .183 50 .152 .090 .052 .056 .209 .183 74 - .071 .050 .056 .209 .133 74 - .071 .050 .001 .003 .133 74 - .050 .001 .003 .133 74 15 14 15 16 17 74 - .001 .001 .003 .00	12 13 14 15 16 17 17 75 .076 .047 .060 .055 .049 75 .076 .047 .026 .047 .063 .063 75 .076 .047 .026 .047 .063 .063 75 .076 .047 .026 .047 .063 .063 75 .079 .076 .071 .060 .106 .063 75 .090 .076 .071 .060 .106 .083 75 .090 .052 .056 .209 .092 .183 75 .17 .16 .17 .16 .17 .13 75 .090 .051 .050 .093 .183 .183 76 .17 .15 .16 .17 .13 76 .071 .050 .003 .004 .003 761 .16 .16 <	13 14 15 16 17 1 .126 .041 .060 .055 .049 .047 .026 .047 .063 .063 .047 .026 .047 .063 .063 .047 .026 .047 .063 .063 .13 14 15 16 17 1 .13 14 15 16 17 1 .052 .056 .209 .092 .133 . .051 .056 .209 .092 .133 . .051 .056 .209 .092 .133 . .052 .056 .209 .092 .133 . .051 .051 .003 .003 .003 . .051 .056 .003 .003 . . .13 .14 .15 051 .14 .15 . </td <td>14 15 16 17 1 26 .041 .060 .055 .049 . 17 .026 .047 .061 .063 .063 17 .026 .047 .061 .063 .049 14 .026 .047 .063 .063 .063 14 15 16 17 1 14 .050 .052 .183 . 25 .056 .209 .092 .183 . 26 .071 .060 .106 .082 . . 26 .002 .003 .092 .093 . . . 20 .000 .003 .003 .003 . . . 21 15 16 20 .003 .003 </td> <td></td> <td>19</td> <td>.042</td> <td>• 043</td> <td></td> <td>19</td> <td>.063</td> <td>960.</td> <td></td> <td>19</td> <td>.003</td> <td>• 006</td> <td></td> <td>19</td> <td>.00</td> <td>.010</td>	14 15 16 17 1 26 .041 .060 .055 .049 . 17 .026 .047 .061 .063 .063 17 .026 .047 .061 .063 .049 14 .026 .047 .063 .063 .063 14 15 16 17 1 14 .050 .052 .183 . 25 .056 .209 .092 .183 . 26 .071 .060 .106 .082 . . 26 .002 .003 .092 .093 . . . 20 .000 .003 .003 .003 . . . 21 15 16 20 .003 .003 		19	.042	• 043		19	.063	960.		19	.003	• 006		19	.00	.010
9 10 11 12 13 14 15 16 17 18 1 .075 .238 .031 .204 .126 .041 .060 .055 .049 .041 .062 .045 .047 .047 .047 .060 .055 .049 .041 .062 .040 176 .047 .047 .047 .043 .054 .054 .062 .210 11 12 13 14 15 16 17 18 .062 .210 .059 .209 .052 .056 .047 .063 .054 .052 .059 .209 .052 .056 .041 .063 .021 .054 .071 .056 .041 .050 .052 .049 .071 .054 .071 .050 .052 .056 .041 .061 .071 .054 .071 .050 .052 .	10 11 12 13 14 15 16 17 18 1 .238 .031 .204 .126 .041 .050 .055 .049 .047 .045 .175 .076 .047 .026 .041 .065 .054 .054 .047 .071 .026 .047 .026 .041 .063 .054 .054 .10 11 12 13 14 15 16 17 18 .17 .201 .039 .203 .056 .205 .205 .133 .027 .203 .152 .090 .052 .205 .209 .14 18 .17 .10 11 12 14 15 16 17 18 .101 11 12 14 15 16 17 18 .101 11 12 14 15 16 17 18	11 12 13 14 15 16 17 18 1 58 .031 .204 .126 .041 .065 .049 .041 55 .076 .047 .026 .047 .053 .054 .054 55 .076 .047 .026 .047 .053 .054 .054 10 .039 .209 .176 .071 .060 .105 .054 .054 10 .039 .209 .176 .071 .060 .105 .054 .054 10 .039 .209 .055 .056 .204 .051 .054 11 12 14 15 16 17 18 .071 11 12 13 .14 15 16 .074 .001 11 12 14 15 15 17 18 .014 11 12 .011 .001	12 13 14 15 16 17 18 1 31 .204 .126 .041 .060 .055 .049 .047 75 .076 .047 .026 .047 .061 .053 .054 75 .076 .047 .026 .047 .041 .063 .054 75 .079 .047 .041 .061 .063 .054 .054 75 .050 .176 .071 .026 .106 .174 18 17 75 .090 .052 .056 .209 .169 .174 18 .027 75 .090 .052 .056 .209 .059 .051 .057 .027 76 .12 14 15 16 17 18 .054 76 .011 .001 .001 .003 .003 .003 .003 71 12 14 <td< td=""><td>13 14 15 16 17 18 1 .126 .041 .060 .055 .049 .041 .047 .026 .047 .041 .063 .054 .047 .026 .047 .041 .063 .054 .047 .026 .047 .063 .054 .054 .176 .071 .060 .166 17 18 .176 .071 .060 .052 .054 .027 .052 .056 .209 .092 .183 .027 .052 .056 .209 .092 .183 .027 .052 .056 .209 .092 .174 .021 .051 .051 .003 .003 .021 .021 .051 .051 .003 .003 .003 .003 .052 .056 .053 .003 .003 .003 .17 .15 .16</td><td>14 15 16 17 18 1 26 .041 .060 .055 .049 .047 26 .041 .060 .055 .049 .047 27 .026 .047 .041 .063 .054 26 .041 .060 .041 .063 .054 26 .071 .060 .106 .052 .174 26 .071 .060 .105 .074 .071 26 .071 .060 .105 .074 .071 27 .050 .002 .105 .074 .071 26 .071 .060 .092 .174 .07 27 .050 .003 .003 .003 .001 27 .16 .17 18 .17 .01 28 .001 .003 .003 .003 .003 .003 29 .001 .001 .003</td><td></td><td>20</td><td>ĩ</td><td>r.</td><td></td><td>20</td><td>.014</td><td>e.</td><td></td><td>20</td><td>ł</td><td>1</td><td></td><td>20</td><td>t</td><td>3</td></td<>	13 14 15 16 17 18 1 .126 .041 .060 .055 .049 .041 .047 .026 .047 .041 .063 .054 .047 .026 .047 .041 .063 .054 .047 .026 .047 .063 .054 .054 .176 .071 .060 .166 17 18 .176 .071 .060 .052 .054 .027 .052 .056 .209 .092 .183 .027 .052 .056 .209 .092 .183 .027 .052 .056 .209 .092 .174 .021 .051 .051 .003 .003 .021 .021 .051 .051 .003 .003 .003 .003 .052 .056 .053 .003 .003 .003 .17 .15 .16	14 15 16 17 18 1 26 .041 .060 .055 .049 .047 26 .041 .060 .055 .049 .047 27 .026 .047 .041 .063 .054 26 .041 .060 .041 .063 .054 26 .071 .060 .106 .052 .174 26 .071 .060 .105 .074 .071 26 .071 .060 .105 .074 .071 27 .050 .002 .105 .074 .071 26 .071 .060 .092 .174 .07 27 .050 .003 .003 .003 .001 27 .16 .17 18 .17 .01 28 .001 .003 .003 .003 .003 .003 29 .001 .001 .003		20	ĩ	r.		20	.014	e.		20	ł	1		20	t	3



tual Coefficients 2 PC

34

			ted	per Column	and per	T ROW for				KAU-Hethod.											
	-	2	3	4	£	9	7	œ	6	10	÷	12	13	14	15	16	17	18	19	20	21
Argression coefficient	1.044	.975	.922	.931 1.038	1.038	.994	1.015	.956	.999 1.179	1.179	686.	1.023	666.	.994	1.019	.994 1.019 1.001	.976		.990 1.010	666.	ï
er column 1	(:025)	(.058)	(110.)	(810.) (210.) (210.) (110.)	(.015)	(.018)	(600.) ((.030)	(.013) (.046)	(.046)	(,00.)	.00%) (.062)	(1:01)		(.019)	.009) (.019) (.011) (.012) ((.014)	(3005)	(110.)	()	ä
Regrecsion coefficient	106.	-917	1.020	.020 1.034 1.221	1.221	1.018	.956	1.029	.995	1.018	1.024	1.006	. 383		195.	.967 1.014	.993	1.012	.979	6	360.1
14 2 2 2 1 1 C	(820.)	(.004)	(200.)	(.045)			(.015)	(.033) ((.013)	(010.)	(.035)	(.024)	(.005)	(200.)	(.016)	(.21.1)	(.013)	(.011)	(.016)	1	(.015
ucrreistion coefficient	• 993	.943	.998	. 995	766.		.999			.932	666.	.941	• 985	.999	•99¢	.993	-997	666.	1.1	.939 1.000	ñ
Correlation coefficient	.942	666.	666.	536.	.994	-992	· 995	.976	.998	. 998	.979	.969	660.	656.	.996	766.	.939	665.	566.	1	156.

tual Coefficients

	The second s	in the second se																			
	F	N	M	4	5	9	7	8	0	10	ų	12	5	14	15	9.	17	18	19	20	53
Regression coefficient	1.067	.929	.963	.972 1.041	1.041	1 566.	1.002	1.030	.986	1.090	066.	.950	.984	.984	-957	5:5	.959	.635	1.005	565.	r
per column i	(.030)	(290.)	(100.)	(.017)	(.023)	(.025)	(.015)	-	(600.	(.035)	(.004)	(.050)	(.056)	(.014) ((.018)	(610.)	(.018)	(800.)	(.cca)	(.002)	3
Fegrezsion coefficient	.913	.952	666.	1.022	.999 1.022 1.110 1.008	1.008	. 934 1	1.032	.1 \$994	1.005	1.035	.985	072.	.995		.963	.835	.995	395	1	1.073
Ther row 1	(650.)	(.006)	(:005)	(.038)	(210.)	(,021)	(.025)	.036)	.005)	(200	(.037)	(.025)	(.006)	(100.)	(.032) ((.015)	(.010)	(200.)	(.021)		(.018)
Correlation coefficient pcz column i	.911	.929	666.	.997	.993				666.	.953	666.	- 955	.952	766.	.994	• 995	• 996	666.	666.	666.	Ĩ.
Lorrelation coefficient	.927	666.	.999	026.	165.	565	.987	.972	666.	666.	086.	195.	666.	656.	585	165.	966.	666.	565.	1	165.

Values of Updated Coefficients and Actual Coefficients Differential petween Results Regression Table 12.

				Lod	per Column and per Row for RAS-method.	and pe	r Row f	or RAS-	nethod.											101 000		
ion Ocefficient .099 .087 .099 .061 .043 .07 .163 .118 .070 .071 .050 .010 .010 .015 .017 .003 .010<			2	30	4	5	9	7	ß	6	10	11	12	13	14		9	17	8	19	20	2:
Imm I	Regression Coefficient	660*	.208	.037	660.	.061			060.	.045	.264	.017	.163	.118	.030	.040			.013	.045	.000	E
ion Coefficient .177 .034 .026 .173 .244 .072 .057 .124 .052 .022 .032 .052 .023 .004 .011 .014 .016 .015 .011 .011 .011 .011 .012 .055 .011	per column i	(.015)	(.036)	(.007)	(.010)	(.011)			(.024)	(600.	.015)	(200.)	(.051)	.017)	(.006)	(710.)	1		.005)	(.005)	(000.)	- 1
1 (.076) (.007) (.007) (.007) (.007) (.015) (.017) (.012) (.012) (.012) (.002) (.021) (.004) (.016) (.011) (.012) (.007) (.011) tion coefficient .601 .727 .927 .820 .627 .522 .628 .961 .725 .763 .644 - .445 .701 - .877 1. tion coefficient .604 .900 .807 .644 - .827 .522 .522 .628 .961 .725 .644 - .445 .701 - .877 1. tion coefficient .664 .990 .807 .441 .957 .623 .523 .188 .586 .365 - .674 - .313 .579 - .869 .745	Regression Coefficient	.177	.034	.026	.138	.244			.107	.022	.032	.124	.052	.023	.002	.033			.046	.059	•	eo1.
tion coefficient .Eo1 .727 .927 .630 .654827 .522 .628 .961 .725 .371 .763 .644445 .701877 unn i coefficient .464 .990 .807 .441 .957 .624 .623 .523 .188 .586 .365674313 .579869 .745	per row :	(.036)	(2003)	(.004)	(.033)	(.016)		1000	(.024)	.012)	.008)	(.022)		(2001)	(200.)	.016) (.005)	(110.)	1	.015)
tion coefficient .464 .990 .807 .441 .957 .624 .623 .523 .188 .586 .365674313 .579869	Correlation coefficient per column i	.501	.727	.927	.830	. 654			.522	.628	-961		.371	.763	.644	r.			ł.		1	1
	Correlation coefficient per row A	.464	.990	6 U	.441	-957	.624			.188		.365	a	•674	Ŧ		-579		.869	.745	e	.539

AC		L
pue	. pout	
Updated	Q. Pmet	
between	low for Q.P.	
Results h	and per F	
noissi	per Column a	
Table 11. 1		
		1

	-	2	10	4	5	9	2	ω	6	10	11	12	13	14	15	16	17	18	19	50	21
tur 10 10 10 - 1	661	000	07A	OFR	1004	960.	.050	.117	.020	.164	.012	.180	.225	.046	.045	.070	.072	.168	.019	.010	1
A MATOTITAON DOISSALBAN	1000	(/	1.00	1000	1110 1			(ver)		(016)	(000)	(1031)	~	(.010)	.024) (.010) (.015) (.012) (.007)	(.012)	(100.)	(.005)	(100.)	(.002)	
per column i	(.019)	.019) (.042) (.	1400-1	1410-1 1610-1 1600	1.0141	1110.1	1600-1	1+20.1	- E	1010-1	TAN-	1.12.1	1								
Degression coefficient	239	-038	.005	.145	131	.076	.114	.134	.012	.019	.132	.065	.037	.015	.082	.037	.169	.020	.064	i	160.
1040444000 +0479010	1 /	1000 1	1100 /	1000 /	1000 1	1000 /		(100)	(000)	(006)	(200)	(1001)	(.003)	(.006)	(.032)	(.015)	(.006)	(.006) (.018)	(.018)	-	(.012)
per row j	(200.)	.0251 (.006) (.		1.0201	1+11-1 1-0201 1-0011 1-0121 1-0121	1210-1		1.001	-+	1222.1	1		1								
Correlation ccefficient per column i	.773	669.	.825	.592	.722	.632	.639	.615	1	.822	1	.637	.875	.553	.282	.659	.371	.992	•	.781	1
Correlation coefficient	.696	.752	5	.676	£16.	.518	.775	.600	.424	.436	.618	.131	.927	1	1	1	-9S7	-557	.244	1	.786

1) Figures between brackets represent standard errors of estimation.