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TUTORIAL PAPER XIX

OPTIMAL CONTROL OF QUEUES: REMOVABLE SERVERS

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ABSTRACT

This paper is devoted to the determination of the optimal operating rule for the behaviour of a removable server. We first examine the case of an individual service process, in M/G/1 queues with N-policy, D-policy and T-policy respectively; certain special cases are also examined such as the finite capacity case, the case of heterogeneous customers,... The problems with batch service are then described and two related applications are considered in details: the control of a shuttle and of a clearing system.

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Queueing systems have already a long life (see "Sixty years in queueing theory", Bhat, Mn. Sc. Vol.15, 6, pp.280) and after the second world war, queueing theory became not only a basic branch of applied probability theory, but also one of the classical methods of O.R. From the outset, some practical problems were treated by queueing systems : e.g., telephone exchange, job-scheduling, etc, ...; nevertheless, queueing theory is often considered by some operations research workers as "a fine mathematical model but ... inapplicable". One of the main reasons of such an opinion certainly is that queueing theory has examined interesting and sophisticated models in the field of applied probability, but often disconnected with decision or optimization problems of O.R. Although some control problems were early introduced in queueing models, they always were static or design problems (in which the system characteristics do not change over time) and clearly this type of problems did not meet the necessities of the greatest part of the practical queueing problems.

In the last twenty years, this situation has considerably changed in particular under the initial impulse, among others, of Naor'school in Israël and research workers, like Heyman, from Bell Telephone Loboratories in U.S.A. There has been an increasing interest in the study of dynamic control problems (in which the system characteristicsare allowed to change over time) and a lot of number of papers concerning this field have been published in the most famous journals of O.R.. One of the main reasons of this development surely is the existence of new practical queueing problems related to the management of great centers, in sectors like distribution, administration, public services, but especially to the management of computer centres. Moreover for the future, the present development of promising research fields, like queueing networks and the performance evaluation of computer networks, create new large possibilities for further applications of control models for more complex queueing systems. (For a stimulating review of such possibilities, see the report "Optimal control of admission to a queueing system" presented by Stidham Jr. at IFORS Congress (augustus 84)).

Several papers have already been published to review this new branch of queueing theory : Crabill-Gross-Magazine /73,77/ wrote a basic survey and a classified bibliography; Sobel /74/, Stidham Jr.- Prabhu /74/, Teghem Jr. /82/ presented others surveys; recently, some sections of Heyman-Sobel's book /82,84/, were devoted to this subject; a forthcoming invited review must soon be published in the European journal of O.R. (Teghem Jr. /85/).
According to the classification introduced by Crabill et al., we can distinguish four main categories of dynamic control models for a queueing system :

I. Control of the number of servers. The servers are removable : they may been turned on or off in function of the state of the system; the varying number of active servers must be determined.

II. Control of the service rate. This category often generalizes the first : the difference is rather than modifying the number of servers, the service process can be changed by varying the service rate.

III. Control of the admission of customers. In these problems, either the arrival rate can be modified, either customers can be refused; in some models the customers control themself the decision to enter into the system.

IV. Control of the queue discipline. Various papers deal with situations where the order of service can be determined. Generally, these problems concern either different classes of customers, either the allocation of customers to different servers.
In this paper, we only treat the first category; it is expected that we present soon, in this journal, other tutorial papers to cover the whole set of these models.

INTRODUCTION

The two first queueing models with a variable number of servers appearing in the literature are those of Romani /57/ and Moder-Philipps Jr. /62/; nevertheless, these papers are descriptive and no cost functions are introduced, a fortiori no optimization problems are setted. So we can consider that the real study of this type of problems begins with the paper of Yadin-Naor /63/. The classical *cost structure* related to a removable server consists in three types of cost :

. r_1 a non negative cost per unit time when the server is on, i.e. in activity; $r_2(\geq r_1)$ a non negative cost per unit time when the server is off, i.e. has decided to not be in activity; let us note $r=r_2-r_1$

. $R_1(R_2)$ a non negative fixed set-up (shut down) cost, incurred each time the server is turning on (off); let us note $R=R_1+R_2$ the total switching cost

. h a holding cost, or customer waiting cost, per unit time and per customer present in the system.

The *problem* consists to determine the optimal operating policies for the removable servers, i.e. to decide when open or close the service channels. It is clear that

. too long keeping a server on involves a too high running cost

. too long keeping a server off involves a too high holding cost

. too much times changing the state of the servers involves too high switching cost.

Thus the optimal policy must correspond to an equilibrium between these three situations.

The *review points*, i.e. the points at which the state of the server can be changed, are

(i) the arrival epochs : the server may be turned on

(ii) the service completion epochs : the server may be turned off.

Note : the hypothesis (ii) can be restrictive : see Heyman /68/, p.369.

The problem is a semi markovian decision process (SMDP) and the two classical criteria for SMDP, with infinite horizon time, can be introduced : either the discounted total cost, with a continuous discount factor $\beta<1$, either the average cost per unit time. The theory of SMDP can be used to prove the existence of a non randomized stationary policy for this problem (see the books "Markovian decision processes" of Mine-Osaki (Elsevier 1970), "Dynamic programming" of Denardo (Prentice Hall 1982), but a special attention must be pointed out to the problem of unbounded costs (see Bell /71/, Lippman /73/, Stidham-Prabhu /74/).

I. A SINGLE REMOVABLE SERVER

Generally, the authors consider the problem in an M/G/1/L queue :

- . L the maximum number of customers present in the system; in the most part of the studies L will be infinite
- . customers arrive according a Poisson process, λ the mean arrival rate
- . the service times are independant identically distributed random variable with distribution function B(.), finite expectation E(S) and finite variance. We denote $\tilde{B}(.)$ the LST of B(.) and $\rho=\lambda.E(S)$; we suppose $\rho<1$ if $L=\infty$.

I.A. *N-Policy*

The most part of the studies characterize the state of the system by the number of customers present and the server applies a N-policy : the state of the server, on or off, will be fixed in function of the number of customers. As preliminaries, let us introduce a particular subset of policies, playing a major part in the following.

Definitions

- . A (ν,N) *policy*, with $0\leq\nu<N<L+1$, consists to turn the server on when N customers are present and turn it off when a service terminates with ν customers left in the system.
- . The *policy* (0,L+1) (or$(0,\infty)$ if $L=\infty$) consists to always close the station.
- . The *policy* (0,0) consists to always open the station.

  Let us suppose that the server applies a (ν,N) policy. The different steady states of the system are
  - (i,0), with $\nu\leq i<N$: the server is off and there are i customers in the queue
  - (i,1), with $\nu+1\leq i<L+1$: the server is on and there are i customers in the system.

We will note

- $p^L_{ik}(\nu,N)$ the steady-state probability that the system is in state (i,k)
- $p^L_i(\nu,N)$ the steady-state probability that there are i customers in the system
- $p^L_{.0}(\nu,N) = \sum_{i=\nu}^{N-1} p^L_{i0}(\nu,N)$ the stationary probability that the server is off
- $N^L_s(\nu,N)$ the mean number of customers present in the system
- $\alpha^L(\nu,N)$ the mean busy period, i.e. a time interval beginning when the station is set up and terminating when for the first time thereafter, the number of customers in the system is equal to ν.
- $n^L_b(\nu,N)$ the mean number per unit time of busy cycles, composed of a successive busy period and idle period (i.e. a time interval during which the server is off).

Note We shall omit the indice L when L=∞

Yadin-Naor /63/ have the first introduced the (0,N) policies in an infinite capacity, proving in particular that

$$N_s(0,N) = N_s(0,0) + \frac{N-1}{2} \tag{1}$$

$$p_{.0}(0,N) = 1-\rho \tag{2}$$

and Teghem Jr. /76/ has established a further relation between policies (0,N) and (0,0) :

$$\begin{aligned} p_i(0,N) &= \frac{1}{N} \sum_{j=0}^{i} p_j(0,0) \qquad i<N \\ p_i(0,N) &= \frac{1}{N} \sum_{j=0}^{N-1} p_{i-j}(0,0) \qquad i \geq N \end{aligned} \tag{3}$$

Loris-Teghem /82/ considered the case of a (ν,N) policy for a finite capacity and obtained, using some results of the theory of regenerative process, that

$$p_{i0}^{L}(\nu,N) = \frac{1}{N-\nu+\lambda\alpha^{L}(\nu,N)} \qquad \nu\leq i\leq N-1$$

$$p_{i1}^{L}(\nu,N) = \frac{\alpha^{i+1-\nu}(0,0)-E(S)}{E(S)(N-\nu+\lambda\alpha^{L}(\nu,N))} \qquad \nu+1\leq i\leq N-1$$

$$p_{i1}^{L}(\nu,N) = \frac{\alpha^{i+1}(\nu,N)-\alpha^{i}(\nu,N)}{E(S)(N-\nu+\lambda\alpha^{L}(\nu,N))} \qquad N\leq i<L \tag{4}$$

$$p_{L1}^{L}(\nu,N) = \frac{(N-\nu)E(S)-(1-\rho)\alpha^{L}(\nu,N)}{E(S)(N-\nu+\lambda\alpha^{L}(\nu,N))}$$

As the mean idle period is obviously equal to $\frac{N-\nu}{\lambda}$, we have by an elementary renewal argument

$$n_b^{L}(\nu,N) = \frac{1}{\frac{N-\nu}{\lambda}+\alpha^{L}(\nu,N)} = \frac{\lambda p_{.0}(\nu,N)}{N-\nu} \tag{5}$$

1. The average case for M/G/1/L

For this criterion, the optimal stationnary policy is independant of the starting state

α) $L=\infty$

Heyman /68/ proves the next property.

Property 1 The optimal policy is either a policy (0,N) with $1\leq N<\infty$, either the policy (0,0)

If C(N) denotes the average cost when the server applies a (0,N) policy, we have for $1\leq N<\infty$.

$$C(N)=r_1 . p_{.0}(0,N)+r_2(1-p_{.0}(0,N)) + R.n_b(0,N)+h.N_s(0,N)$$

and using (1), (2) and (5).

$$C(N)=r_1 + r_2(1-\rho) + \frac{R\lambda(1-\rho)}{N} + h(N_s(0,0) + \frac{N-1}{2}).$$

For policy (0,0) we have

$$C(0) = r_2 + h\, N_s(0,0)$$

As C(N) is a convex function of N, Heyman /68/ concludes that the optimal value of N is either N=0, either one of the two integers surrounding the value

$$N^* = \sqrt{\frac{2\lambda R(1-\rho)}{h}} \qquad (6)$$

Remarks

(i) Three papers investigated this problem in the more general GI/G/1 queue. Sobel /69/ obtains sufficient conditions on more general cost structure for the existence of an optimal (ν,N) policy; Heyman-Marshall /68/ give bounds on the cost function and the optimal policy in the case of interrarrival distribution with increasing rate. Applying a method of diffusion approximation, Kimura-Ohno-Mine /80/ characterize the average cost rate and give some sufficient conditions under which the optimal operating policy falls into specific forms.

(ii) Talman /79/ gives another proof of the optimality of a (0,N) policy.

(iii) Yadin-Naor /63/ have also introduced the notion of set-up time, i.e. a random interval ellapsed before the service station is really reactived when such a decision is taken. For an M/M/1 queue, Baker /73/ analyses the consequences of an exponential set-up time on the value N^*.

β) $L<\infty$

Hersh-Brosh /80/ and Teghem Jr. /84/ have investigated the case $L<\infty$; in their model with limited capacity, the holding cost h is replaced by a penalty cost c_L incurred for every lost customer. When $L<\infty$, it is not more true that the policy (0, L+1) is never optimal. Using (4) and (5), Teghem Jr. /84/ extends and generalizes the results of Hersh-Brosh /80/ proving the next property :

Property 2

Consider the plane $(\bar{r} = \frac{r}{c_L}, \bar{R} = \frac{\lambda R}{c_L})$; it is divided in (L+2) regions corresponding to the optimality of policy (0,N), N=0,...,L+1, as showed in figure 1

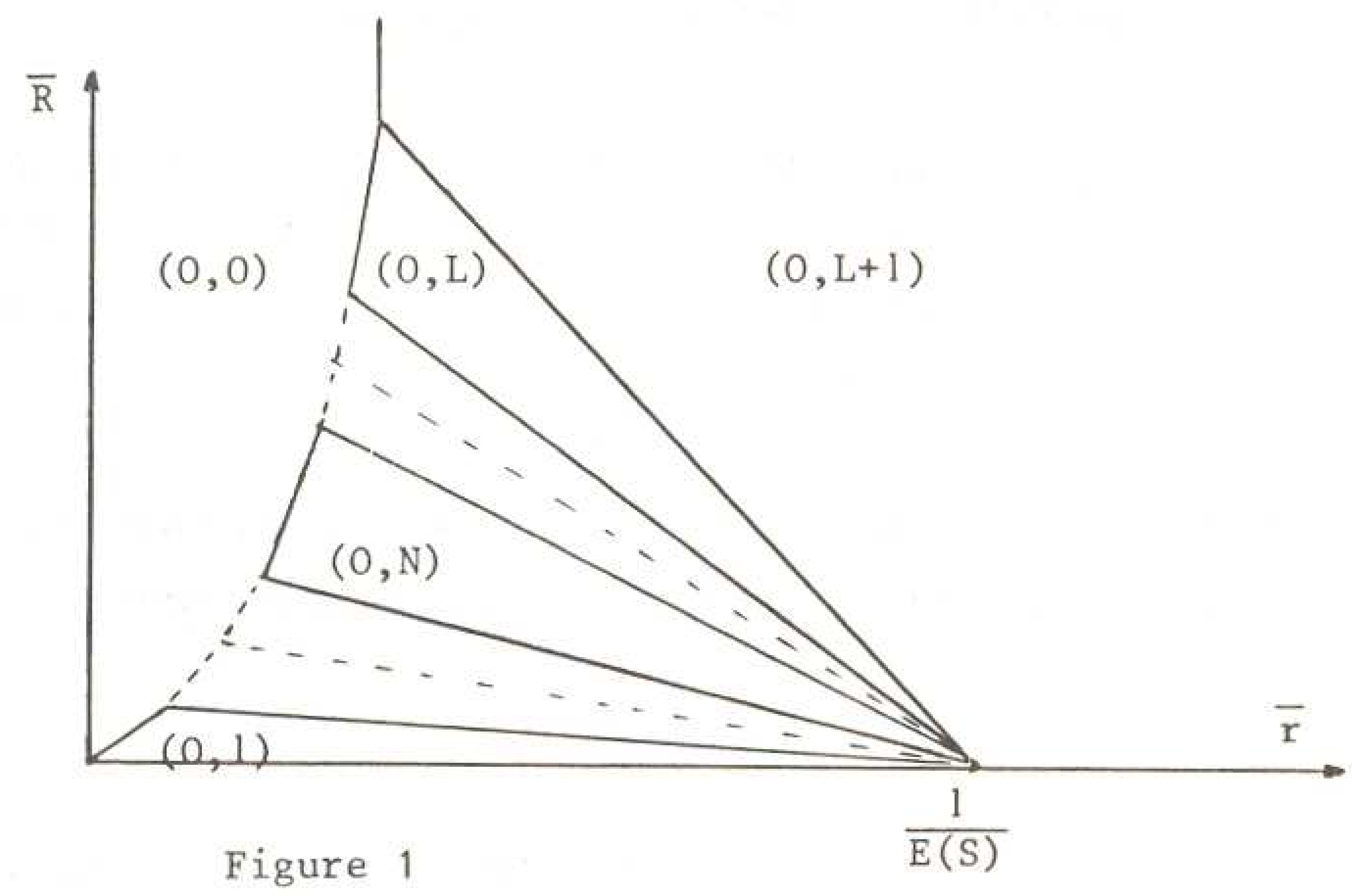


Figure 1

Moreover the equations of the frontiers of each region are explicited; they are also determined by the points of coordinates (r_N, R_N) with

$$r_n = \frac{1}{E(S)} \left(1 - \frac{1 + \lambda\, \alpha^{L-N}(0,0)}{1 + \lambda \alpha^{L}(0,0)}\right)$$

$$R_N = \sum_{j=0}^{N-1} (r_N - r_j)$$

Remarks

(i) The interaction between the optimal operating rule of a removable server in a finite capacity M/M/1 queue and the optimal behaviour of customers are simultaneously analysed in Teghem Jr. /77/

(ii) Bidhi Singh /82/ study a M/M/1/L queue, wherein, when the queue length increases to N(0<N<L), a search for an additional service facility for the service of a group of units is started; the availability time of this additional service facility is a random variable, but the search is dropped when the queue length reduces to some tolerable size ν. The optimal (ν,N) policy is investigated for a cost structure (with R=0) for this additional removable server.

2. The discounted case for M/G/1

For this criterion, the optimal stationary operating policy depends on the initial starting state; for easyness, let us suppose that this starting state is (0,0).
Heyman /68/ and Bell /71/ obtain the property

Property 3 — The optimal policy is either a policy (0,N), with $0 \leq N \leq \infty$, either a policy $(\overline{0,N})$, with $1 \leq N < \infty$, consisting to turn the server on at the first time when N customers are present and never off again.

Let us note $C_\beta(N)$ and $\overline{C}_\beta(N)$ the total discounted case, respectively for policies (0,N) and $(\overline{0,N})$. It is easy to determine

$$C_\beta(\infty) = \frac{r_1}{\beta} + \frac{h\lambda}{\beta^2}$$

but the determination of $C_\beta(N)$ and $\overline{C}_\beta(N)$ is more difficult. Heyman /68/ and Bell /71/ obtain

$$C_\beta(N) = (R_1 \tilde{A}_N + \frac{r_1}{\beta}(1-\tilde{A}_N) + \frac{r_2}{\beta} \tilde{A}_N(1-\tilde{G}_N) + R_2 \tilde{A}_N \tilde{G}_N + H(N))(1-\tilde{A}_N \tilde{G}_N)^{-1}$$

$$\overline{C}_\beta(N) = R_1 \tilde{A}_N + \frac{r_1}{\beta}(1-\tilde{A}_N) + \frac{r_2}{\beta} \tilde{A}_N(1-\tilde{G}_N) + \overline{H}(N)$$

where

. $\tilde{A}_N$ is the LST of the distribution of an idle period

$$\tilde{A}_N = (\frac{\lambda}{\lambda+\beta})^N \text{ for } N \geq 1 \quad \text{and} \quad \tilde{A}_0 = \frac{\lambda}{\lambda+\beta}$$

. $\tilde{G}_N$ is the LST of the distribution of a busy period, determined by

$$\tilde{G}_N = \tilde{G}(\beta)^N \quad \text{for } N \geq 1 \quad \text{and} \quad \tilde{G}_0 = \tilde{G}(\beta)$$

with $\tilde{G}(\beta)$ determined by the implicit equation

$$\tilde{G}(\beta) = \tilde{B}(\beta+\lambda-\lambda\tilde{G}(\beta)) \tag{7}$$

. $H(N)$ and $\overline{H}(N)$ are the terms corresponding to the holding costs.

Unfortunately, the expression of H(N) and $\overline{H}(N)$ are more complicated (see Bell /71/ p.210 and appendix) and, like $\tilde{G}(\beta)$,given by (7), almost impossible to calculate explicitely. Thus in practice, it is quite difficult to determine the optimal operating policy by the algorithm provided by Bell /71/. In order to deal with this difficulty, Kimura /81/ considers a diffusion approximation model depending only on the first two moments of the distribution function B(.)and derives approximation formula for $C_\beta(N)$ and $\overline{C}_\beta(N)$.

Remarks

(i) Blackburn /72/considers the same model, but with balking and two different types of reneging (single and batch reneging); for this case of impatient customers, he generalizes the results of property 3.

(ii) Langen /76/ gives a different form of the costs $C_\beta(N)$ and $\overline{C}_\beta(N)$.

3. Finite source M/G/1

α) Jaiswal-Simha /72/ consider a server applying a(0,N) policy in a finite source queue M/G/1 (i.e. each unit stays in the source for a random time exponentially distributed with parameter λ); let us note I the size of the source. In this model, which can be interpreted as a repair shop for I machines, the holding cost h is replaced by a reward g per unit time for each running machine, thus for each customer not present in the queueing system. These authors treat discounted, as well as undiscounted criterion; we give here, for instance, the average case.
Let P(N) be the total expected profit per unit time when the server applies a (0,N) policy; with an evident extension of notation, it is described by
$P(N) = g(I-\overline{N}_s(0,N)) - R.\overline{n}_b(0,N) - r_1.\overline{p}_{.0}(0,N) - r_2(1-\overline{p}_{.0}(0,N))$
when, for this finite source model, these authors obtain

$$\overline{N}_s(0,N) = I - \frac{1}{\lambda E(S)}(1-\overline{p}_{.0}(0,N))$$

$$\overline{n}_b(0,N) = \frac{\overline{p}_{.0}(0,N)}{\sum_{j=0}^{N-1} \frac{1}{(I-j)\lambda}}$$

with a specific value of $\overline{p}_{.0}(0,N)$ (see formula 22, p.701, Jaiswal-Simha /72/). Some numerical examples are treated for the determination of the optimal value of N.

β) We will introduce in this section a quiet different but interesting model concerning a simple closed queueing network. Hatoyama /77/ considers a *discrete time maintenance system* with I machines and two stations : an operating and a repair facility. The model is illustrated in figure 2 :

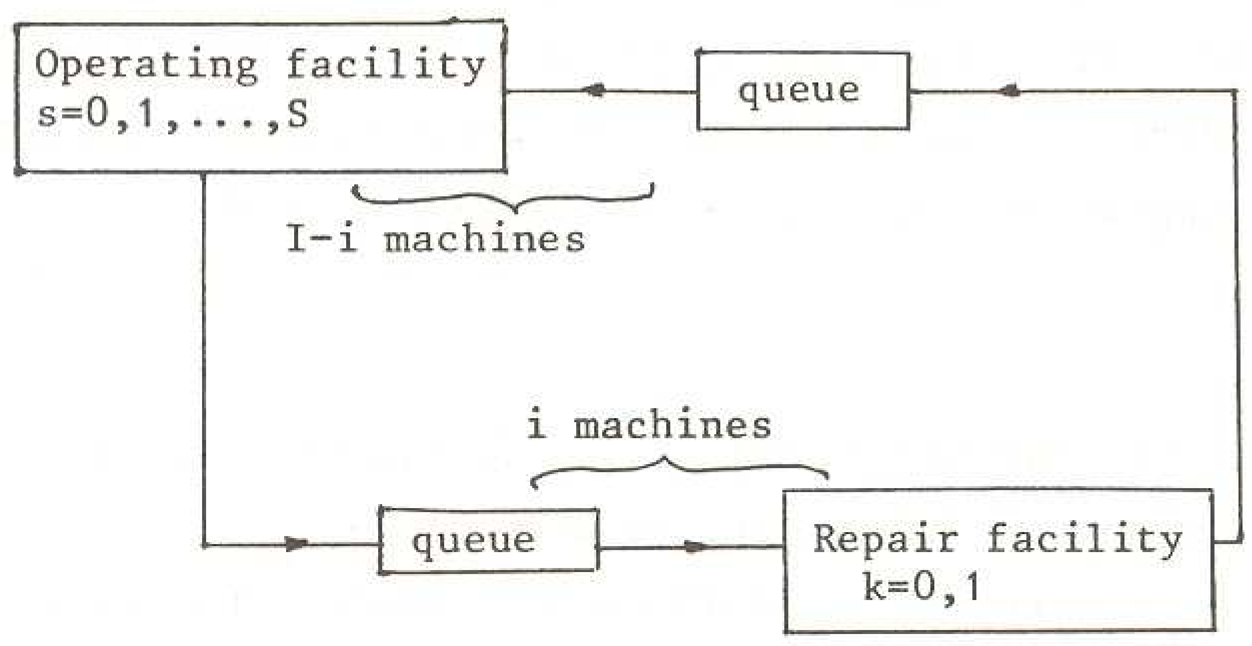


Figure 2

. Operating station : at the beginning of each period, an operating machine is classified as being in one of S+1 states (s=0,...,S), showing the degree of deterioration (0 : best state; S : failed state). An operating machine evolves from state s to state s' in one period , according to a transition probability $p_{ss'}$. An operating machine can be sent to the repair shop at any period and is then instantly replaced by a spare unit, if any available.

. Repair station : a machine sent to the repair shop must wait until all the machines which have already arrived at the repair shop are completely repaired; moreover, at the beginning of each period, the decision maker has the option of opening or closing the repair shop. When the repair station is open and there are i machines, q_{ij} is the probability that j of these machines are still in the repair system at the end of the period.
At the beginning of a period, the state of the system is thus described by (i,k; s) with
- i : the number of machines at the repair shop (i=0,...,I)
- k : equal to zero (one) if the repair shop is closed (open)
- s : the state of the machine at the operating station (when i<I).

This author associates with this system the following costs :

. Repair station : fixed switching costs R_1 and R_2; running cost r_1 per period; a general holding cost H(i,k) per period, depending of the state of the station.

. Operating station : an operating cost a(s) per period for a machine in state s; a reparing fixed cost c(s) for a machine in state s; a penalty cost P per period when no operating machine is available.

Hatoyama /77/ derives sufficient conditions on this structure cost for the existence of an optimal *two dimensional control-limit* policy : a control limit policy

- with respect to operating station : $\forall (i,k), \exists S_{(i,k)} \succ \forall s<S_{(i,k)}$ it is optimal to leave the operating machine and otherwise to repair it
- with respect to repair station : $\forall (s,k), \exists I_{(s,k)} \succ \forall i<I_{(s,k)}$ it is optimal to close the repair shop and otherwise to open it.

4. Several classes of customers

α) For the average case, Bell /73/ studies the optimal behaviour of a removable server in an M/G/1 priority queue, with two types of customers (k=1,2) having identical service time distribution, but characterized by different arrival rates λ_k and holding costs h_k. Customers of class 1 have non preemptive priority over customers of class 2 and $h_1 \geq h_2$.
Bell /73/ proves the property 4.

Property 4 — The optimal policy is either a policy $(0, N(i_1, i_2))$ consisting of turning the server off when the system is empty and turning the server on when i_1 or i_2, the number of customers of each class, reaches or crosses a linear boundary of the form $c_1 i_1 + c_2 i_2 + d = 0$; either the policy (0,0).

This author establishes that the line $c_1 i_1 + c_2 i_2 + d = 0$ is included, like in figure 3, between the two lines $i_1 + i_2 = N_k^{**}$ (k=1,2) corresponding to the cases of a same holding cost h for each customer, respectively equal to h_1 and h_2; moreover its slope is always smaller than -1 and equal to -1 only when $h_1 = h_2$.

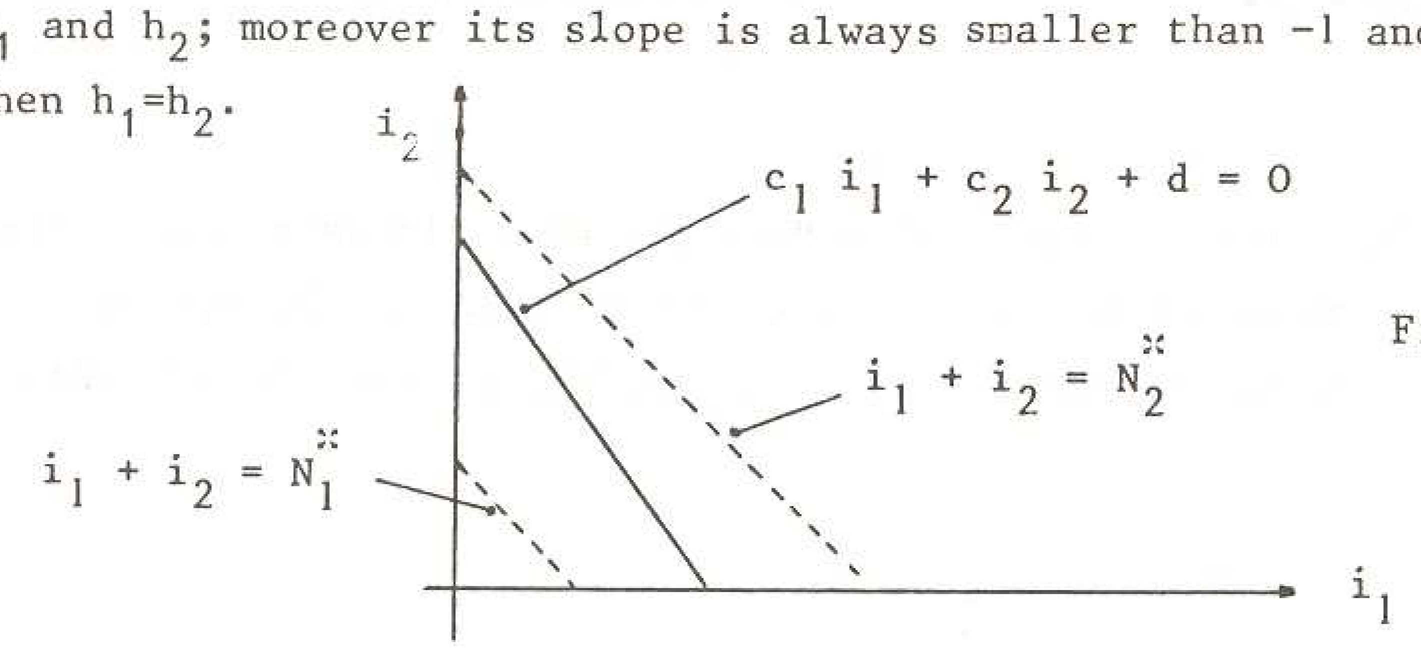


Figure 3

Let us note that Tijms /74/ extends these results to the case of different service time distributions for each type of customers and derives, using some results of the theory of regenerative process, expression for the average number of customers, of each class, present in the system.

β) In his Ph.D.Ghorayeb /78/ considers the same type of model but without any assumption concerning the priority and with switching costs, also to move the server from one class to the other. The operating policy must then determine not only when to open or to close the station, but also which class of customers to serve and when to change to the other class. This general problem seems really difficult and the author introduces some limitative assumptions : the main one is that there are no arrivals of class 2 when the server is busy. Ghorayeb /78/ proves property 5.

<u>Property 5</u>

There exists an optimal policy represented by figure 4 in the plane (i_1, i_2) and such that :

. in area I : if the server is off or on, he remains off or on

. in area II : if the server is off, he is turned on and he begins to serve first the customers of class 1

. in area A : if the server is on, he continues to serve the same type of customer

. in area B : if the server is on, he always serves customers of class 1.

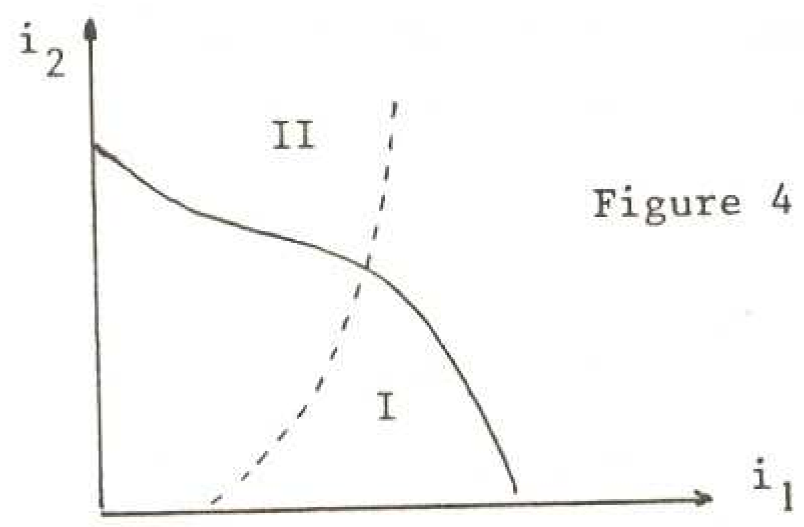
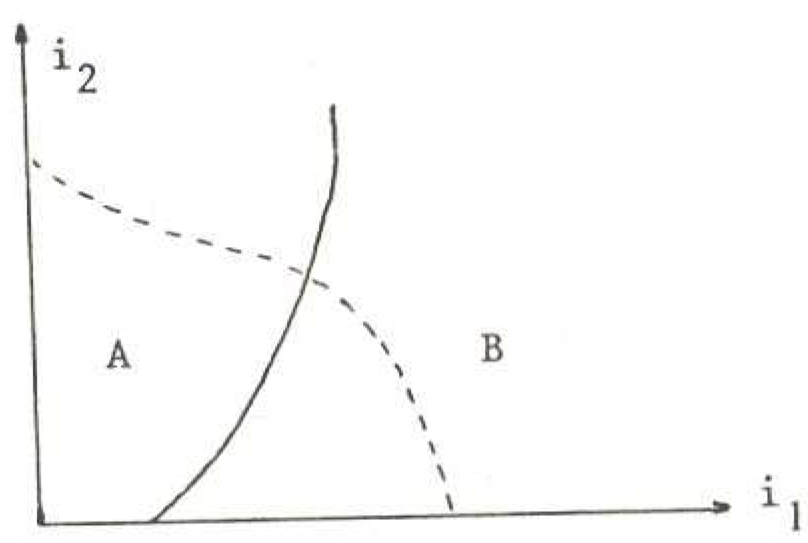


Figure 4



I.B. *D-Policy*

Balachandran /73/ has the first introduce the model in which the state of the system is the workload i.e. the total amount of work in the system. The idea is that the customer's service times are different even though they may come from the same distribution; yet this type of measure means that service times must be known immediately, after the customer enters the queue. A(0,D) policy consists then to turn on the server when the total work to be done reaches the value D and turn him off when the system is empty. For the average cost criterion, this author analyses the (0,D) policies for a similar cost structure as in I.A., except that $r_1=r_2=0$ and h is now a holding cost per unit time per unit work. If C(D) represents the average cost, we have like in I.A., and with an evident extension of notations,

$$C(D) = R.n_b\,(0,D) + h\,W(0,D)$$

where W(0,D) is the expected work in the system.
Balachandran-Tijms /75/ and Tijms /76/ obtain

$$n_b(0,D) = \frac{\lambda(1-\rho)}{E(M_D)}$$

$$W(0,D) = W(0,0) + D - \int_0^D \frac{E(M_x)}{E(M_D)}\,dx$$

where M_x represents the number of customers present at the opening of the station if a (0,x) policy is applied and W(0,0) is the average waiting time in an M/G/1 system with policy (0,0); these authors derive D^* the value corresponding to the minimum of C(D)

$$D^* + \int_0^{D^*} E(M_x)\,dx = \frac{R\,\lambda(1-\rho)}{h}$$

For the cost C(D), the policy $(0,D^*)$ is compared with the policy $(0,N^*)$. For a constant service time, the two policies are obviously equivalent. Boxma /76/ generalizes results of Balachandran-Tijms and proves the optimality of the D policy over the N policy. Note that Tijms /77/ gives an expression of the stationary distribution of the workload, when a policy (0,D) or a policy (0,N) is applied.

I.C. *T-Policy*

1. FIFO discipline

Levy-Yechiali /75/ consider an M/G/1 queue, with usual FIFO discipline

("First in first out"), such that when the server finishes serving a unit and finds the system empty, he goes away for a length of time called a vacation. At the end of the vacation, the server returns to the main system and begins to serve if they are customers. If the server finds the system empty at the end of a vacation, two models are introduced :

Model 1 : the server waits for the first customer to arrive and then an ordinary busy period begins

Model 2 : the server immediately takes another vacation and continues in this manner until he finds at least one waiting unit upon return from a vacation.

As usual, the server serves the queue as long there is at least one unit in the system.
F(.) denotes the distribution function of the random vacation T, with finite mean E(T) and second moment $E(T^2)$. The same cost structure than in section I.A. is introduced, except that $r_2 < r_1$ and in fact $r=r_1-r_2>0$ represents the reward per unit time of the server for the work done during the vacation.

By evident extension of notation, the average profit per unit time for models j=1, 2 respectively, is equal to

$$P^{(j)}(T) = r\ p_{.0}^{(j)}(0,T) - R.n_b^{(j)}(0,T) - h.N_s^{(j)}(0,T)$$

Let we note

$$\gamma = \frac{\lambda E(T)}{f_0+\lambda E(T)} \qquad \text{with } f_0 = \int_0^\infty e^{-\lambda t}dF(t)$$

Levy-Yechiali /75/ obtain

$$p_{.0}^{(1)}(0,T) = \gamma.p_{.0}^{(2)}(0,T) \qquad \text{with } p_{.0}^{(2)}(0,T) = 1-\rho$$

$$n_b^{(1)}(0,T) = \frac{\gamma}{1-f_0}\ .\ n_b^{(2)}(0,T) \qquad \text{with } n_b^{(2)}(0,T) = \frac{(1-f_0)(1-\rho)}{E(T)}$$

$$N_s^{(1)}(0,T) - N_s(0,0) = \gamma(N_s^{(2)}(0,T) - N_s(0,0))$$

with $\quad N_s^{(2)}(0,T) - N_s(0,0) = \dfrac{\lambda E(T^2)}{2E(T)}$

so that $\quad P^{(1)}(0,T) = \gamma\left(P^{(2)}(0,T) - R\,\dfrac{(1-\rho)f_0}{E(T)}\right)$

These authors conclude that, for a fixed distribution F(.), model 2 is superior to model 1 if and only if

$$p^{(2)}(0,T) > -\lambda(1-\rho)R;$$

they also determine the expressions of an optimal value of T (his expected value) for deterministic (exponential) vacation times.

Note As $p._0^{(2)}(0,T)$ is independant of T, there is no need to introduce the cost r in model 2.

Heyman /77/ examines model 2; when the server finds the system empty at the end of a vacation, he considers that a busy period of length zero occurs and that the cost R is thus incurred; in that case we have

$$n_b^{(2)}(0,T) = \frac{1-\rho}{E(T)}$$

and for deterministic vacation times, the optimal value of T is

$$T^{*} = \sqrt{\frac{2\ R(1-\rho)}{\lambda h}} = \frac{N^{*}}{\lambda}$$

This author compares the average cost for this optimal T policy and the optimal N policy and proves that the latter always does better than the former.

Remarks

(i) Meilijson-Yechiali /77/ consider a priority control model in a GI/G/1 queue, in which insertion of idle time is allowed.

(ii) Van der duyn Schouten /78/ introduces a descriptive model with stochastic vacation time and a finite capacity for the workload and derives several characteristics : the joint stationary distribution of the workload and the stage of the server; the average number of overflows per unit time and the average number of vacations per unit time.

(iii) Note that some queueing problems in which the service station is subject to breakdown are close of the removable server model.

2. SPT discipline

α) Three papers have been more recently published by Shanthikumar /80[a],80[b],81/; note that his results can be applied as well to N-policy that to T-policy, but we only present the latter. In the first paper, the author develops a new and interesting method to analyze some controlled M/G/1 queueing problems, using properties of the number of up and downcrossings levels in a special case of regenerative process. He obtains two important basic relations between the density and the expected number of upcrossings of this regenerative process (see formula 8 and 9, p.817, Stanthikumar /80[a]/); these equations can be used in many queueing systems,especially with exponential arrivals. For instance, Stanthikumar /80[a]/ uses this method to easily derive the results of Levy-Yechiali /75/ concerning the virtual waiting time distribution for T-policy.

β) By this method, Shanthikumar /80[b]/ analyses optimal T-policy (model 2) of a server in an M/G/1 queue with shortest processing time (SPT) discipline : at the service completion epochs, the server choosesto serve the customer with the shortest service time. For this model, let us note W(SPT; T) the expected waiting time of an arbitrary customer; W(FIFO; T) may be determined by relation (8) and Little formula.
Shanthikumar /80[b]/ determines the LST of the waiting time distribution; then he obtains W(SPT; T) and proves the next conservation identity :

$$\frac{W(FIFO;T)}{W(SPT;T)} = \frac{W(FIFO;0)}{W(SPT;0)} \qquad \forall\ T \qquad (10)$$

In the case of deterministic vacation time, he derives the optimal value of T

$$T^{*}(SPT) = T^{*}(FIFO)\ .\ \sqrt{E} \qquad (11)$$

where $T^{*}(FIFO)$ is given by (9) and E is the value of identity (10).

γ) Shanthikumar /81/ applies the same procedure for a different, but quite close, queueing discipline, called SPT within generations.
(SPT-WG) : the customers that arrive during the vacation form the first generation and their total service time is the lifetime of the generation; customers arriving during the lifetime of the first generation, if any, make up the second generation, with its lifetime, and so on; within each generation, customers are served in the order of the SPT discipline.
This author obtains similar results as (10) and (11), i.e. with obvious extension of notation

$$\frac{W(FIFO;T)}{W(SPT\text{-}WG;T)} = \frac{W(FIFO;0)}{W(SPT\text{-}WG;0)} = E' \qquad \forall\ T$$

and

$$T^{*}(SPT\text{-}WG) = T^{*}(FIFO)\ .\ \sqrt{E'}$$

II. MULTI REMOVABLE SERVERS

The problem of more than one removable server is only investigated in a few studies. A basic difference is that when the unique server is turned off, the queue size necessarily must increase, but when one of several servers is turned off, the queue size may go up and down. Mc Gill /69/ was the first to examine this problem and established some intuitive properties for the form of optimal policies, in the case of a general discounted cost GI/G/1 queueing system, but only for a finite horizon. Bell /75/ considers this problem for an infinite horizon M/M/S model; a classical cost structure is considered and fixed switching costs are incurred to turn each server on or off. The state of the system is now denoted (i,k) when there are i customers and k servers working. This author calls *efficient policy*, an operating rule which never allows more working servers that customers present; otherwise the policy will be called inefficient. He proves that for r sufficiently high and all others parameters fixed, there exists an efficient policy; otherwise an optimal policy may turn one or more servers off, even when there are customers for him to serve, i.e. may be inefficient. Bell /80/ further investigates this model for S=2 and first shows that a critical number N exists such that all the servers should be turned on or left on in any state (i,k) with $i \geqslant N$. Generalizing (ν,N) policies of section I, he definies a (ν_1,ν_2,N_1,N_2) policy for which the 4 critical levels denote numbers of customers in the system when the number of working servers should be adjusted downward to 0,1 and upward to 1,2 respectively. For R=0, obviously an optimal policy adjusts the number of working servers to min {i,k}, i.e. $\nu_1=0$, $\nu_2=N_1=1$, $N_2=2$. If R is allowed to increase from 0, he obtains the following property.

Property 6 — The best efficient policies is such that $\nu_1=0$, $1<\nu_2$. Yet, if an inefficient policy is optimal, it may be of three types :

- leave both servers on at all times ($\nu_1=\nu_2=N_1=N_2=0$)
- leave at least one server at all times ($\nu_1=N_1=0$)
- decrease the number of servers only in state (0,2) and turn off both servers ($\nu_1=\nu_2=0$).

Remarks

(i) Magazine /69/ and Huang - Brumelle - Sawaki - Vertinsky /77/ consider control models under periodic reviews, i.e. the review points are at equally spaced time intervals.

(ii) Levy-Yechiali /76/ consider T-policies in an M/M/S queueing system and derives formula for the distribution of the number of busy servers and the mean number of units in system.

(iii) Winston /78/ examines several removable servers in an exponential queueing system in which the arrival rate depends on the number of customers. For state (i,k), a general holding cost h(i) and a running cost r(k) are introduced, but no switching costs. This author derives conditions that ensure the optimality of monotone policies such that the number of working servers is a non decreasing function of the number of customers in the system.

III. *BATCH SERVICE AND RELATED AEREAS*

III.A. *Batch service*

An interesting problem concerns a removable server who can make a decision to serve any number of customers in a batch, up to some batch size limit $1 \leq Q \leq \infty$. For this model, Deb /76/ introduces the following cost structure :

. r_1, r_2, R_1, R_2 like before
. h(i) : a general non linear cost for holding i customers during a unit time
. c.y a linear cost for serving y customers (we have y=min(i,Q)).

The approach of this author is different in the sense that he establishes the form of an optimal policy by direct analysis of the infinite horizon functional equation of SMDP.

Let us introduce relations (11) and (12) respectively for the discounted and average criterion :

for some $\eta>0$, $\quad h(i) - h(i-1) > \eta \qquad (12)$

$$h(i) - h(i-1) > \frac{1-\tilde{B}(\beta)}{\tilde{B}(\beta)Q}(\beta R_1+r) + \frac{\beta}{\tilde{B}(\beta)} c + \eta \qquad (13)$$

Property 7

a) If (12) and (13) hold, respectively for the two criteria, there exists an optimal policy of the form (ν,N)

b) Otherwise the policy $(0,\infty)$ of turning the server off for ever is optimal.

Remark

In the particular case $R_1=R_2=0$, Deb-Serfozo /72/ proves that for property 7.a), we have $\nu=N-1$, i.e. there exists an optimal policy of the type control limit policy. For this case and moreover with $Q=\infty$, Weiss /79,81/ presents some properties of the cost-function, gives an algorithm for finding the optimal control limit and determines the waiting time distribution. Finally, let us note that Weiss-Pliska /82/ introduce a general holding cost $h_t(i)$ depending of time and show that control limit policies may cease to become optimal.

III.B. *Control of a shuttle*

Batch service queueing systems are often found in transportation, since mass transit vehicles are natural batch servers; so a related application of model described in III.A. is the optimal dispatching of a shuttle. Let be a shuttle system consisting of a single carrier with capacity Q, transporting passengers between two terminals. At each terminal, passengers arrive according to independant Poisson processes (λ_1,λ_2) and all arriving passengers wait to be transported to other terminals where they exit the system; B(.) is here the distribution of the interterminal travel times, independant of everything and in particular of the load carrier. The system is reviewed at those points in time when, either the carrier has just arrived at one of the terminals or when the carrier is waiting at one of the terminals and a new passenger arrives. The state of the system is denoted by (i_0, i_1, δ), where i_0, i_1 are the number of passengers at the two terminals and δ, equal to zero or one, indicates at which terminal, 0 or 1, is the carrier. At each review point, it is necessary to decide if the carrier is dispatched (with $\min(i_\delta, Q)$ customers) or not. The cost of carrying y passengers is R + c.y and there is a linear holding cost h.

1. Control at both terminals

Some particular cases have been first considered. Ignall-Kolesar /72/ study the case Q=1 and moreover the dispatch decision is made without knowledge of the queue at other terminal; the paper of Barnett /73/ concerns deterministic travel times with some restrictions on the values λ_1 and λ_2. Barnett-Kleitman /78/ show that the result for the control at a single terminal (see below) is not directly generalized for control at both terminals. The general model is introduced by Deb /78/. In a first time, he considers the finite horizon period n and extends then the results for $n \to \infty$; for the discounted criterion, he proves the following property.

Property 8

a) If $h < \beta(c + \frac{R}{Q})$, then the policy of never dispatching the server is optimal

b) Otherwise, the optimal control policy is of the form : Dispatch the carrier iff $i_\delta \geq G_\delta(i_{1-\delta})$, where $G_\delta(.)$ is a monotone decreasing control function.

Unfortunately, the explicit determination of the function $G_\delta(.)$ seems an unsolved problem.

2. Control at one terminal

Ignall-Kolesar /74/ examine the particular problem of the control at a single terminal - said zero - and for an infinite capacity shuttle with deterministic travel times. Thev prove the following property :

Property 9 — There exists an optimal control limit policy of the form : Dispatch the carrier iff $i_0 + i_1 \geqslant N$.

Weiss /81/ presents a method for computing the control limit N, compares this policy with the more traditional policy of scheduled periodic service and last, proves a conjecture of Ignall-Kolesar /74/ regarding the case when the dispatcher does not know the number of passengers at terminal 1 : there exists an optimal control policy concerning i_0 plus the expected number of passengers at terminal 1.

Remarks

(i) Osuna-Newell /72/ and Asgharzadeh-Newell /78/ consider a particular model of multiple vehicle system.

(ii) Teghem Jr. /82/ consider a double shuttle system, like a ropeway, transporting passengers simultaneously from one terminal to the other in the two opposite directions.

III.C. *Clearing systems*

Stochastic clearing systems are first analysed by Stidham Jr. /74/ and optimized by the same author in 77.
The cumulative input to such a system is described by a non decreasing stochastic process $\{Y(t), t \geqslant 0\}$, with $Y(0)=0$; output occurs intermittently in the form of clearing operations, which instantaneously remove all the quantity in the system. This author considers that clearing occurs whenever the cumulative input since the last clearing instant exceeds a critical level q. Let us introduce some definitions and notations.

- X_1, time until first clearing; X_n, time between $(n-1)^{th}$ and n^{th} clearings $(n>1)$
- $S_0=0$, $S_n = \sum_{j=1}^{n} X_j$, $n \geqslant 1$

. $R(t) = \max\{n | S_n \leqslant t\}$, the number of clearings in $[0,t]$

. $V(t) = Y(t) - Y(S_{R(t)})$, the net quantity in the system

. $T(y) = \inf \{t | Y(t)>y\}$, the first entrance time into the set (y,∞)

. $W(y) = E(T(y))$, the sojourn measure of the set $[0,y]$.

With

Assumption 1 $\{V(t),t \geqslant 0\}$ is a regenerative process with respect to the renewal sequence X_n, $n \geqslant 1$.

Stidham Jr. /74/ obtains the stationary distribution of $\{V(t),t \geqslant 0\}$: it is completely defined by knowledge of $W(y)$, $\forall y \leqslant q$ and is different that the stationary distribution uniform between 0 and q.

Stidham Jr. /77/ introduces the following costs :

. a positive cost R - independant of q - whenever a clearing takes place

. a general holding cost $h(x) \geqslant 0$, incurred while $V(t)=x$

and

Assumption 2 h is continuous and - h is unimodal with mode x_0 $\varepsilon(-\infty,\infty)$. Let we note $h'(x) = h(x_0+x)$.

The average cost C(q) is given by

$$C(q) = \frac{R + \int_0^q h'(x-x_0)\, dW(x)}{W(q)}$$

and this author proves.

Property 10 Let $\tilde{q}$ be a solution to the equation $\int_0^q W(x)dh'(x-x_0) = R$ (14)
Then $\tilde{q}$ minimizes $C(q)$ among all $q \geqslant 0$, such that $W(q)>0$
(If there is no solution to (14), then $\tilde{q}=\infty$).

Rather than "N-policy", Nishimura /79/ considers T-policy in this model. He first obtains an optimal clearing interval $\tilde{T}$ among the set of non negative random variables with finite mean (see theorem 3.3., p.101, Nishimura /79/) and then proves that if $h(x)$ is continuous and monotone non decreasing in $x \geqslant 0$, then $\tilde{T} = T(\tilde{q})$.

These two authors Stidham Jr. /77/ and Nishimura /79/ generalize their results to a general clearing system in which the effect of a clearing operation is that the quantity in the system is restored to a level ν rather than 0.

Last, let us note that Whitt /81/ further investigates the comparison between the stationary distribution of V(t) and the uniform distribution and Stidham-Serfozo /73/ introduce more general clearing systems in which, in particular, the quantities cleared are random variables.

CONCLUSION

We have here examine some of the principal papers related to removable servers; yet we have of course no claim to be exhaustive. It is important to remark that there is no major difficulty to classify in categories the different papers because, unfortunately, very few studies consider the optimization of more than one parameter. It seems us important in the future to analyze the interactions between several different optimization problems. We invite the reader, interested by further comments on the prospects of the field of optimal control queueing problems, to refer to the forthcoming paper of Teghem Jr. /85/.

To conclude, we want to turn the attention of the reader on the possibility to determine an optimal policy of very complex optimal control problems - for which it can not be expected that the optimal policy has simple form - by using numerical algorithms issued of the SMDP theory. A good example of this technic is given by the paper of K.Ohno-K.Ichiki /84/ ("An optimal control problem of a C-stage tandem queueing system" Technical report-Department of Information processing and Management Sciences, Faculty of Science, Konan University, Kobe, Japan). It is important to not forget this type of approach to resolve complex practical problems.

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