TRANSPORTATION TECHNIQUE FOR QUADRATIC FRACTIONAL PROGRAMMING

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ABSTRACT. — In this paper a transportation technique for a quadratic fractional programming subject to linear constraints has been provided. In another article it has been shown that this function is pseudo monotonic which gives local optimum as global optimum.

Introduction

This paper deals with a special type of problem which occurs in big business concerns to fill a number of vacancy categories which demand different capabilities, experiences and trainings. The applicants having different capabilities, experiences and trainings will have the value depending upon the jobs in which they are to be employed. It is always the sincere intention of the concern to assign the applicant categories to vacancy categories in such a way that the value of the objective function with which the business concern is dealing with is a maximum. The objective function considered here is pseudo monotonic [2].

This paper is the outcome of the main results of the paper [2] in which the author has proved that maximum will occur at the vertex of the feasible solution set and local maximum is global maximum. The present paper has been divided into three sections. In section 1, mathematical model is given. Preliminaries are given in section 2. Section 3 deals with the optimal conditions.

Section 1

Mathematical Model.

Maximize

$$f(x) = \frac{\left(\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \alpha\right)^2}{\left(\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} x_{ij} + \beta\right)^2} \qquad (1)$$

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subject to

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j = 1, 2, ..., n \tag{1.2}$$

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1, 2, ..., m \tag{1.3}$$

$$x_{ij} \geqslant 0 \qquad i = 1, 2, ..., m \qquad j = 1, 2, ..., n \tag{1.4}$$

where x_{ij} = set of structural variables; these variables represent competitive candidates or activities.

c_{ij}, d_{ij} = set of profit coefficients in the problem and are the coefficients of the structural variables in the objective function.

Section 2

Preliminaries.

(i) The consistency condition for the existence of the solution to the problem is

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

In case $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$, then a ficticious personnel categorry $\sum_{j=1}^{n} b_i - \sum_{i=1}^{m} a_i$ men is added to the problem. When $\sum_{j=1}^{n} b_j < \sum_{i=1}^{m} a_i$, then a ficticious job category containing $\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$ jobs is used [3].

(ii) These are in all $m + n$ equations in (1.2) and (1.3), out of which always only $m + n - 1$ are non-redundant i.e. any basis will involve only $m + n - 1$ variables [4].

(iii) The set of feasible solutions is regular and non-empty.

(iv) Initial basic feasible solution can be found by using one of the well-known methods : North-West Corner Method, Volga's Method and Inspection Method [6].

(v) Simplex Multipliers. As in [7], we determine the simplex multipliers p_i^1, p_i^2 $(i = 1, 2, ..., m)$ and q_j^1, q_j^2 $(j = 1, 2, ..., n)$ from equations

$$\left.\begin{aligned} c_{ij} + p_i^1 + q_j^1 &= 0 \qquad (2.1) \\ d_{ij} + p_i^2 + q_j^2 &= 0 \qquad (2.2) \end{aligned}\right\} \; i, j \text{ take suffixes of basic variables}$$

Further let

$$\left.\begin{aligned} c_{ij}^1 &= c_{ij} + p_i^1 + q_j^1 \\ d_{ij}^1 &= d_{ij} + p_i^2 + q_j^2 \end{aligned}\right\} i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n \qquad \begin{aligned}(2.3)\\(2.4)\end{aligned}$$

It must be noticed that we are dealing with a system of $m + n - 1$ equations out of $m + n$ equations given in (1.2) and (1.3) as one equation is always redundant. The choice of the redundant equation is immaterial, we may set arbitrarily one of the p_i or one of the q_i equal to zero and solve for the remaining $m + n - 1$ simplex multipliers. These simplex multipliers would be unique as the set of equations (1.2) and (1.3) are independent. We shall make use of these values of the simplex multipliers in (2.3) and (2.4) to determine c_{ij}^1 and d_{ij}^1 for the non-basic variables.

Section 3

We shall determine here the next best basic feasible solution which improves the value of the objective function. Objective function would be written in terms of non-basic variables only. The function $f(X)$ is

$$f(X) = \frac{\left(\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \alpha\right)^2}{\left(\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} x_{ij} + \beta\right)^2}$$

$$= \frac{\left[\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \sum_{i=1}^{m} p_i^1 \left(\sum_{j=1}^{n} x_{ij} - a_i\right) + \sum_{j=1}^{n} q_j^1 \left(\sum_{i=1}^{m} x_{ij} - b_j\right) + \alpha\right]^2}{\left[\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} x_{ij} + \sum_{i=1}^{m} p_i^2 \left(\sum_{j=1}^{n} x_{ij} - a_i\right) + \sum_{j=1}^{n} q_j^2 \left(\sum_{i=1}^{m} x_{ij} - b_j\right) + \beta\right]^2} \qquad (3.1)$$

because of $\sum_{j=1}^{n} x_{ij} = a_i$ and $\sum_{i=1}^{m} x_{ij} = b_j$ whatever may be the values of p_i^1, p_i^2, q_j^1, q_j^2.

(3.1) can also be put in the form

$$f(X) = \frac{\left[\sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij} + p_i^1 + q_j^1)\, x_{ij} - \sum_{i=1}^{m} p_i^1 a_i - \sum_{j=1}^{n} q_j^1 b_j + \alpha\right]^2}{\left[\sum_{i=1}^{m} \sum_{j=1}^{n} (d_{ij} + p_i^2 + q_j^2)\, x_{ij} - \sum_{i=1}^{m} p_i^2 a_i - \sum_{j=1}^{n} q_j^2 b_j + \beta\right]^2} \qquad (3.2)$$

p_i^1, p_i^2, q_j^1, q_j^2 are chosen such that

$$\left.\begin{array}{l} c_{ij} + p_i^1 + q_j^1 = 0 \\ d_{ij} + p_i^2 + q_j^2 = 0 \end{array}\right\} \begin{array}{l} i, j \text{ take suffixes of the basic set} \\ \text{of } m+n-1 \text{ basic variables.} \end{array}$$

$$f(X) = \frac{[\sum_{i,j \in S} c_{ij}^1 x_{ij} - \sum_{i=1}^{m} p_i^1 a_i - \sum_{j=1}^{n} q_j^1 b_j + \alpha]^2}{[\sum_{i,j \in S} d_{ij}^1 x_{ij} - \sum_{i=1}^{m} p_i^2 a_i - \sum_{j=1}^{n} q_j^2 b_j + \beta]^2} \tag{3.3}$$

where S is a set of non-basic variables. Making use of the given basic feasible solution, the value of the objective function at that basic feasible solution becomes

$$\frac{[-\sum_{i=1}^{m} p_i^1 a_i - \sum_{j=1}^{n} q_j^1 b_j + \alpha]^2}{[-\sum_{i=1}^{m} p_i^2 a_i - \sum_{j=1}^{n} q_j^2 b_j + \beta]^2} = \frac{[T_1]^2}{[T_2]^2} \tag{3.4}$$

where

$$T_1 = -\sum_{i=1}^{m} p_i^1 a_i - \sum_{j=1}^{n} q_j^1 b_j + \alpha$$

$$T_2 = -\sum_{i=1}^{m} p_i^2 a_i - \sum_{j=1}^{n} q_j^2 b_j + \beta$$

All the non-basic variables are zero at the initial basic feasible solution.

Equation (3.3) can be rewritten as

$$f(X) = \frac{[\sum_{i,j \in S} c_{ij}^1 x_{ij} + T_1]^2}{[\sum_{i,j \in S} d_{ij}^1 x_{ij} + T_2]^2}$$

Now we choose x_{pq} variable to enter the basic set at a value W (> 0) and x_{lm} [7] variable departs from basic set i.e. becomes non-basic at zero level. The value of objective function becomes [1] $\bar{f}(X)$ such that

$$\bar{f}(X) = \frac{[T_1 + W\, c_{pq}^1]^2}{[T_2 + W\, c_{pq}^2]^2}$$

In case $f(X) > \bar{f}(X)$ (strictly) {when all basic feasible solutions are non degenerate} i.e.

$$\frac{[T_1 + W\, c_{pq}^1]^2}{[T_2 + W\, d_{pq}^1]^2} - \frac{[T_1]^2}{[T_2]^2} > 0 \tag{3.6}$$

we can say that the value of the objective function will improve.
(3.6) becomes

$$[\mathrm{W}^2 (c_{pq}{}^1)^2 + 2\,\mathrm{T}\,\mathrm{W}\, c_{pq}]\, \mathrm{T}_2{}^2 - [\mathrm{W}^2 (d_{pq})^2 + 2\,\mathrm{T}_2\, \mathrm{W}\, d_{pq}{}^1]\, \mathrm{T}_1{}^2 > 0$$

(denominator of the objective function is positive always) or

$$[\mathrm{T}_2\, c_{pq}{}^1 - \mathrm{T}_1\, d_{pq}{}^1]\, [\mathrm{W}\, \{\mathrm{T}_2\, c_{pq} + \mathrm{T}_1\, d_{pq}\} + 2\,\mathrm{T}_1\, \mathrm{T}_2] > 0$$

c'_{pq} and d'_{pq} refer to the original basic feasible solution. Let

$$\phi_{ij} = [\mathrm{T}_2\, c_{pq}{}^1 - \mathrm{T}_1\, d_{pq}{}^1]\, [\mathrm{W}\, \{\mathrm{T}_2\, c_{pq}{}^1 + \mathrm{T}_1\, d_{pq}{}^1\} + 2\,\mathrm{T}_1\, \mathrm{T}_2]$$

and it can be calculated for non-basic variables if $p_i{}^1$, $p_i{}^2$, $q_j{}^1$, $q_j{}^2$ are known for the same.

Here $\phi_{pq} = \max \phi_{ij}$ $(\phi_{ij} > 0)$ i.e. we choose the most positive ϕ_{ij} to determine the variable x_{pq} to enter the basis. The variable x_{lm} which is to leave the basic set and the value of the basic variables in the new basis can be determined in the same way as in the case of transportation problem in linear programming.

In case all $\phi_{ij} \leqslant 0$ we get the optimal solution of the given problem.

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