A TAXONOMY OF ECONOMICALLY BASED QUALITY CONTROL PROCEDURES

Ehud Menipaz, Ph.D., P.E.
Concordia University, Montreal

Not long ago, industrial engineers moved to acquire more insight into the area of economic considerations in quality control. Contributions in this area fell into four basic categories: procedures for the economic selection of a quality level, procedures for the designing of optimal sampling plans, procedures for the design of a sequence of sampling plans, and procedures for process control.

Since a satisfying literature review of this area cannot be found, this presentation is concerned with a taxonomy and descriptions of currently existing economically based quality assurance procedures.

Major existing procedures are mentioned and current and future procedures are described in detail. The choice of the procedures for detailed examination is made both on the basis of the availability of information and with an aim to obtain an adequate coverage of approaches. The evaluation is in terms of:

a) data requirement - how many different factors have to be measured and evaluated? What is the degree of difficulty of measuring and evaluating these factors? b) Built-in assumptions - what stated and unstated assumptions are built into the procedures? c) Usefulness of outputs - is the procedure directly applied within a context of production process by those in charge of the quality control function? d) Versatility - can the model be used in different contexts or is it limited to one specific use? Among other things, the taxonomy analyzes: data requirements, manual or computer programmes necessary to execute the calculation of different parameters, difficulty in updating, operator proficiency level, the costs that are taken into account, the possibility of using the procedure in the context of assemblies.

The Nature of the Survey

While a most important facet of industrial quality control is the economic design and evaluation of quality control procedures, very little work of a general nature has been published on this problem.

Two basic categories of operational quality control procedures may be identified:

a) Sorting procedures, that is, procedures intended to separate between good and defective products with respect to some quality characteristics.

b) Process control problems, that is, procedures intended to maintain the quality level of a process at some desired, predetermined quality level.

One can find statistical characteristics and non-statistical characteristics in quality control. The latter include the technical problems of measurement (reliability, validity and speed), and the economic design and economic evaluation of quality control procedures. This survey describes a number of different type models that have appeared in the literature which deals with this subject.

The survey categorizes the different models in the following manner:

a) Models for the economic selection of a quality level.

b) Models for the selection of the optimal sampling plans.

c) Models for the design of consecutive sampling plans.

d) Models for process control.

The first three categories deal with the sorting procedures, while the last set of models deals with the process-control problem. Figure 1 presents the different categories that were mentioned above.

INSERT FIGURE 1

Each of the following sections will be devoted to a descriptive analysis and brief discussion of a particular model.

Important existing models are mentioned and current models are described in detail. The choice of the model for detailed availability of information and with an aim to obtain an adequate coverage of approaches.

The evaluation is in terms of:

a) Data requirements - how many different factors have to be measured and evaluated? What is the degree of difficulty in measuring and evaluating those factors?

b) Built-in assumptions - what stated and unstated assumptions are built into the model?

c) Usefulness of outputs - is the model directly applied within a context of production process by those in charge of the quality control function?

d) Versatility - can the model be used in different contexts or is it limited to one specific use?

Generally, models dealing with the economic aspects of quality control have been developed in the context of manufacturing processes. This explains the frequent use of terms like "incoming lots" and "outgoing lots" in the context of such models.

Models for the Economic Selection of a Quality Level

The various models in this category dealt with the following: given the various sampling plan tables: (1), (2) select the sampling plan which minimizes an objective function that takes into account the costs involved in accepting a lot with many defectives and the costs involved in rejecting a lot with fewer defectives. By and large, the average proportion of defective units, i.e., the average defective ratio, in the incoming lot was assumed known. The sampling plan tables were built in such a way that once a plan had been chosen, the quality of the outgoing lots was determined.

Among the models noteworthy in this context are those by Enell (3), Martin (4) and Shahnazarian (5).

Enell. Enell's model (3) presented an economic approach to the problem of determining the requirements for a sampling plan.

It dealt chiefly with the choice of an Average Quality Level (AQL) for Military Standard 105A (1), and it was probably the most widely used table of attribute sampling plans.

This model dealt with the case of acceptance sampling. In acceptance sampling the alternatives were to accept or to reject the lot received. When the decision was to accept, one had to consider the costs that were involved in dealing with defectives that were present in the lot that was accepted. A lot which was rejected would normally be sorted and defectives found, repaired or replaced. The total of these costs was the price of rejection.

Through the survey, definitions of costs and other terms are not the ones that have appeared in the source cited. An effort has been made to use the same symbols throughout the survey in order to achieve unity. Symbols that are peculiar to any specific source are defined in the first place they appear.

Let

k_8 = unit cost of acceptance (the cost incurred when a defective piece slipped through into subsequent production operations)

k_1 = cost of inspecting one piece (good or bad)

k_{11} = cost of repairing or replacing a defective component once found

p = the (unknown) fraction of defectives in the lot

k_9 = unit cost of rejection, i.e., cost of finding a defective in a rejected lot, plus expense of correcting it.

The model found the breakeven point, p_b, i.e. the fraction of defectives in the incoming lot for which the expected loss due to acceptance of the lot was equal to the expected loss from its rejection.

The breakeven point was

$$k_8 = k_9 = \frac{k_1}{p} + k_{11} \qquad [2.1]$$

$$p_b = \frac{k_1}{k_8 - k_{11}} \qquad [2.2]$$

If the cost to replace the defective component, k_{11}, was small compared to the damage done by a defective which slipped through into the production process, k_8, then the relation [2.2] became approximately

$$p_b = \frac{k_1}{k_8} \qquad [2.3]$$

Then, according to the model, one had to refer to the operating characteristic curve in the Military Standard 105A (1) in order to choose the suitable plan.

The economic analysis pointed out that the sampling that one chose, on the average,should accept lots that had less than the p_b per cent of defectives and, on the average, should reject lots that had a per cent defective greater than p_b. This implied that one had to choose a curve which passed near the probability .50 at p_b per cent defective. The various plans were characterized by the relation between the proportion of defectives and the probability of acceptance, using the Average Quality Level (AQL) as a parameter.

The model had useful features. It had the advantage of being simple while taking into account the main factors that should affect the decision. Nevertheless, this model was limited to the economic choice of AQL's for attribute sampling. Besides, the crucial factor in deciding whether to use sampling for a specific case was not the average per cent of defectives, but rather the variability of that per cent of defectives from lot to lot. According to the model, if the quality was substantially better than breakeven and was also stable, no inspection was needed. If it was substantially worse than breakeven, and consistently so, it was least expensive to use one hundred per cent inspection without stopping to sample. If the quality was erratic, then sampling paid for itself.

Enell's model (3) seemed to take care of this point but not in a direct way. No relations had been developed that would indicate the changes in the AQL as a function of the variability of the per cent defective in an incoming lot or how close should this value have to be to p_b to justify sampling.

Martin. A scheme had been developed by Martin (4) whereby cost estimates were used to compute a critical per cent of defectives. This per cent of defectives was used to make a decision whether to accept lots with superficial inspection for surveillance and observation of the average per cent of defectives, or to impose a tighter plan.

Let

c = number of defectives in sample

n = size of sample

N = size of the lot from which the sample is taken

$P_a(p)$ = probability that c defectives or less would be found

($P_a(p)$ was a function of p which was the proportion defective of incoming lot).

Two cases of attribute sampling were treated: non-destructive sampling and destructive sampling.

Non-Destructive Sampling

To derive a breakeven point, the author compared the combined cost of inspection, replacement and acceptance of a defective unit, with the cost of no inspection, for different proportions of defectives p.

If

$$N' = 1 - \frac{n}{N} \qquad [2.4]$$

then

$$\text{Combined Cost} = k_1 (1 - P_a N') + k_9 p (1 - P_a N') + k_8 p P_a N' \qquad [2.5]$$

$$\text{No-inspection Cost} = k_8 p \qquad [2.6]$$

Equating [2.5] and [2.6] gave

$$k_1 (1 - P_a N') - k_9 p (1 - P_a N') + k_8 p P_a N'$$

$$= k_8 p \qquad [2.7]$$

[2.7] reduced to

$$p_b = \frac{k_1}{k_8 - k_9} \qquad [2.8]$$

This relation was useful for comparing several sampling plans on a single chart and also for finding the upper and lower bounds for the cost of each p.

The following steps were suggested for determination of a non-destructive sampling:

a) Estimate the elements of cost

b) Compute the $p_b = \frac{k_1}{k_8 - k_9}$, or $\frac{k_1}{k_8}$, if k_9 is not used.

c) If the process average plus three standard deviations (or whatever band appears suitable from the observed variance) lies left of the p_b (see Figure 2), do not inspect at all or sample only for proper surveillance, and to maintain process average data.

d) If the process average plus three standard deviations lie right of the p_b (see Figure 2), use the loosest plan whose $P_a = 0$ on the operating characteristic curve in the neighbourhood of the prevailing process average plus three standard deviations.

INSERT FIGURE 2

<u>Destructive Sampling</u>

Let

$v.k_1$ = the cost of screening examination of a rejected lot (non-destructive)

$w.k_1$ = the cost of destroying one unit in the process of destructive sampling

$s.n$ = the size of the second destructive sampling (n was the size of the original sample) (The replacement cost did not exist in this kind of sampling).

Cases of destructive sampling procedures were investigated. In the first case a sample of n was tested to destruction. When c or less defects were found,the lot was accepted and was forwarded. When more than c rejects were found, the lot of N was rejected. (N-n) were then examined non-destructively at a cost of $v.k_1$ per unit, and the defective condition was corrected. Then, as a check, a destructive inspection was made of a sample of s.n size. The screened lot was assumed to be in a satisfactory condition. The expected cost for this case was

$$C_1 = nk_1(1 + w) + vk_1 (N - n) (1 - P_a)$$

$$+ snk_1 (1 + w) (1 - P_a) + k_8 pP_a (N-n) \qquad [2.9]$$

The expected cost of no inspection was $k_8 p$. Equating those costs yielded the breakeven ratio of defects, p_{bd} .

$$P_{bdl} = [\frac{k_1}{k_8}][(1 - N') (1 + w) + vN'(1 - P_a)$$

$$+ (1 - N') s (1 + w) (1 - P_a)] / (1 - P_a N')$$

when

$$N' = 1 - \frac{n}{N} \qquad [2.10]$$

The second case that was treated was the case where, as before, each rejected lot was screened but there was no second inspection. The lot of size N, less the initially destroyed n, went forward, s = 0

$$C_2 = k_1 [(1 - N') (1 + w) + vN' (1 - P_a)] + k_8 pP_a N' \qquad [2.11]$$

$$P_{bd2} = [\frac{k_1}{k_8}] [(1 - N') (1 + w) + vN' (P_a)] / (1 - P_a N') \qquad [2.12]$$

The third case was when the rejected lot was scrapped; there was no screening and no cost for the scrapped unit, v = s = 0

$$C_3 = k_1 (1 - N') (1 + w) + k_8 pP_a N') \qquad [2.13]$$

$$P_{bd3} = [\frac{k_1}{k_8}] n(1 + w) / [N - P_a (N - n)] \qquad [2.14]$$

The fourth case was the same as the third case, but when a cost of scrapped unit was included

$$C_4 = k_1 \left[(1 - N') (1 + w) + wN' (1 - P_a)\right] + k_8 p P_a N' \qquad [2.15]$$

$$P_{bd_4} = \left[\frac{k_1}{k_8}\right] \left[w + (1 - N') / (1 - P_a N')\right] \qquad [2.16]$$

Unlike the case of a non-destructive sampling, each sampling plan had its own breakeven point, as seen in Figure 3.

INSERT FIGURE 3

For each plan, the p_{bd} was a function of P_a , but P_a was further a function of p itself. As the values of p increased, the p_{bd} function of p usually decreased until the values coincided; then the difference grew as p increased and p_{bd} reached a constant level.

Martin treated both the case of destructive and non-destructive sampling. The breakeven point approach for the non-destructive sampling case was valuable for the lot method with one hundred per cent screening of rejected lots. In the non-destructive sampling plan, when the process average laid to the left of the breakeven point (see Figure 2), light inspection was indicated. When the process average laid to the right of the breakeven point (see Figure 2), a tight inspection was indicated. This removed guesswork for selecting a sampling plan, when the average defective ratio in the process was known. The formula for the non-destructive breakeven point was simple to derive.

The expressions of the breakeven function for destructive sampling, on the other hand, left much to be desired as a criterion. The user had to examine the various sampling plans using the cost curves plotted against the line of no inspection, or p (see Figure 2).

<u>Shahnazarian</u>. While Enell and Martin studied the economics of sampling versus no sampling, the work by Shahnazarian (5) dealt with the economics of sampling versus one hundred per cent inspection of each incoming lot.

Basically, the objective of Shahnazarian's paper was to find the defective ratio of the incoming lots, for which one was indifferent between one hundred per cent sorting of the components and sampling inspection that allowed some defective components to enter the assembly operation. This sometimes was referred to as the breakeven defective ratio.

Once the breakeven point had been established, one could use it to determine the most economical plan for the specific product arrangement. If the quality level could be controlled at breakeven fraction defective or better by the sampling plan - it would be the most economical plan.

The mathematical model was developed for the case where components were received or internally produced by the vendor or a preceding department. Upon receipt at the user's facility, components were checked by a sampling plan, and with acceptance, continued their flow through the plant. The assumption was that if at least one defective component was assembled, then the assembly became defective for repair or scrap.

For control purposes, the quality level was established as the maximum quality level tolerated. Beyond this point it was more economical to perform one hundred per cent inspection. Therefore, the supervisor executed a sampling plan (6) based on the Average Outgoing Quality Limit, thus defining the maximum level that was tolerated.

Let

$C(k_1)$ = cost function of sorting components

$C(k_7, k_8)$ = cost function of having defective components enter the assembly

p^* = fraction defective of components entering assembly

q^* = fraction of good components entering assembly, $q^* = 1 - p^*$

L = number of components per assembly

X' = number of defective assemblies

k'_{11} = cost of assembly scrap per piece

n' = number of assemblies inspected

k'_1 = assembly inspection cost per assembly

The model found the breakeven point by equating

$$C(k_1) = C(k_7, k_{10}) \qquad [2.17]$$

$$k_1 N = k'_{11} X' + k'_1 n' \qquad [2.18]$$

since

$$X' = n' - n' q^L \qquad [2.19]$$

therefore,

$$\log q = \frac{1}{L} \log \left[1 - \frac{k_1 N - k'_1 n'}{k'_{11} - X'} \right] \qquad |2.20|$$

which expressed the breakeven point, q^*. The meaning of q^* was the following: if the quality level could be controlled at this fraction of defectives, the most economical plan for the process has been determined.

The author further discussed the effect of varying the number of components per assembly on the breakeven quality level. The effect was merely of increased cost of scrapping the assembly as the number of components increased and, thus, a lower breakeven point resulted. A decrease of the breakeven point has been noticed by the author under automatic inspection programs for isolating defective components because the unit inspection cost per component would be less than inspection using manual methods.

The model was useful for establishing quality levels that had a known effect on quality costs. The two alternative quality programs that were weighted were one hundred per cent sorting of components,versus a sampling plan that allowed defective components in assembly.

Note that the models that have been presented in this section have taken the following approach: given the various sampling plan tables: (1),(6), select the sampling plan which minimizes an objective function that takes into account various cost components.

Models for the Design of Optimal Sampling Plans

Models of this sort dealt with the designing of a sampling plan, based on various cost components, i.e., what would be the critical fraction of defectives; what would be the sample size; etc... In this context, some earlier and current approaches are noteworthy, (7), (8), (9), (10), (11), (12), (13). Earlier models are mentioned and recent models are given a more thorough description.

Theoretical consideration of some cost components affecting a given process or product was given by Wald (14) in his formulation of risk functions, which included the cost of errors and the cost of sampling.

Satterthwaite had derived expressions for minimum total cost single (15) and continuous (16) attributes sampling plans for the case of a known proportion of defectives in two successive quality lots being submitted. The cost equations which were minimized are fairly complete, but even if the cost data were available, economic design of quality control procedures was proved to be a complicated procedure using this model.

Breakwell (17), (18), developed a model for the computation of single and sequential acceptance plans, including both attributes and variables which were normally distributed, which minimized the maximum value of a Wald-type risk function of the form:

$$C(k_1, k_8, k_9) = \begin{cases} K_8 (p - p_c) p_a + nk_1, & p > p_c \\ K_9 (p_c - p) P_r + nk_1, & p < p_c \end{cases}$$

[2.21]

where

$C(k_1, k_8 k_9)$ = the risk function

p_c = the critical fraction of defectives

$k_8 (p - p_c)$ = the loss due to acceptance of lot with $p > p_c$

$k_9 (p_c - p)$ = loss due to rejection of lot with $p < p_c$

nk_1 = sample number which equals a measure of the sampling cost

Others concerned with the economic facet of quality control were Duncan (19) and Cowden (20). They considered the economic balance of sampling costs, costs of wrong decisions and general operating costs in determining the sample size, the sampling interval and the control limits for minimum-cost control of a stochastic process. Cowden's cost equation was extremely restrictive, however. For a significant number of sets of conditions, both Breakwell (18) and Cowden found the best solution as being no inspection at all. In the following sections, recent models in this category are described in more depth.

Hald. Hald (9) developed a system of single sampling inspection plans based on prior distributions.

Let $f_N(X)$ = prior density function of X defectives in lot of size N.

It has been stated that if $f_N(X)$ was hypergeometric, binomial, Polya or any weighted average of these where the weights did not depend upon N and X, then defectives x found in the sample of n had the same distribution as X with n substituted for N, i.e., $f_N(X)$ could be reproduced by sample selection. Hald's paper was the most exhaustive article in this family of models. A single sampling plan was defined by three parameters (the lot size N, the sample size n, the acceptance number c) and the following decision rule: acceptance of the lot if the number of defectives in the sample was equal to or less than the acceptance number; otherwise, the lot was rejected. His model gave a range of sampling plans which were useful for different prior distributions of defectives, $f_n(x)$. So far as implementation of Hald's model was of concern, two cases had to be distinguished: a) the case where the data that pointed to the kind of prior distribution was available, i.e., if the parameters in a Polya distribution or a binomial distribution could be estimated - the determination of the optimum plan was straightforward; and b) the case where no data was available but only a partial knowledge of the prior distribution; one had to guess a limiting distribution and to choose the optimum plan accordingly. A lir ting distribution was one which described the maximum possible proportion of defectives in a lot. Hald's results were practical even for case b) above, since it was believed that it was possible to estimate the distribution's parameters out of a discussion between the engineer, the inspector and the statistician as the basis for determining a sampling plan. As information from sampling inspection accumulated, it has been used at regular intervals to test and estimate changes of the prior distribution.

Heermans. Heermans' model (10) determined the optimal in-process sampling plans for minimizing the total sampling and defective product costs. Formulation of the model followed a Bayesian approach which was related to the approach of Hald (9) and Guthrie and Johns (8). Heermans used a two-step search method for deriving the optimal sampling plan. At this point one should mention the work by Smith (13) that developed a model for sampling plan selection based on models by Guthrie and Johns (8). But, unlike Heermans (10), Smith has solved his model analytically.

Penkov and Theodoresen. Penkov and Theodoresen (12) developed a most interesting application of game theory to the interpretation of the role of economic sampling for quality control. They derived rules for the estimation of the minimum sample size required. In the sampling context the producer was regarded as a player opposed to nature. Nature controlled a continuous set of pure strategies p (i.e., the fraction of defectives) in the interval $[0, 1]$. The producer had only two strategies available: the acceptance x_A and the rejection x_R of the products. Two cases were treated, a case where no sampling has been executed and a case where $c - \Sigma[\underline{f}, \overline{f}]$ was the acceptance number

$\underline{f}$ = the lower limit of Z

$\overline{f}$ = the upper limit of Z

Guenther. Guenther (21) developed a procedure for easy determination of sampling plans, based on Hald's linear cost model (9) and prior distribution. The ideas were based on the linear cost model suggested by Hald (9). The main merit was that the procedure of determining the sampling plans was easy, using the process' fraction of defectives.

Models for the Design of A Sequence of Sampling Plans

These models (22), (23), (24), (25), (26), presented a description and analysis of the interacting effects that exist between inspection stations which were arranged in a sequential series along a production line. In contrast with the models described earlier, this approach takes into account the fact that the choice of sampling plan for a given stage determined not only the costs incurred at that stage, but also the quality of material available to all subsequent manufacturing and inspection stages. A sequence of sampling plans which was truly optimal for an entire system could be found by treating each station in isolation from all others, since savings could be made for some stations at the expense of increased costs at later stages. That explains the need for models which are quite different from the models that have been described earlier.

Beightler and Mitten. Beightler and Mitten (27) proposed a description and analysis of the interacting effects that existed between sequentially arranged quality control stations. An optimal sequence of sampling plans was the sequence defined by that set of parameters which minimized the total cost. The minimization was for a given average incoming lot quality vector, known transition costs and fixed values for the conditional probabilities. For that given set of parameters, the total expected cost, K, was a function only of the sampling plan parameters. Two different computation procedures were suggested: one used the gradient method and the other used a dynamic programming formulation.

Lindsay and Bishop. This article (27) dealt with a method of determining minimum-cost allocations of screening effort to satisfy both a quality requirement and linear cost function of outgoing defectives. Since the computational procedure allowed rapid desk-calculator solutions to this problem, the methods which have been described should prove useful to those who may be faced with the problem of allocation of screening procedures, but lack the availability of large scale computing equipment. The goal was to determine stagewise sampling levels in a multistage process which minimized the total of sampling and scrap costs. The decision variables in this model were the inspection levels at the various stages of the process. The final determination of the minimum cost inspection program directed to achieve a required quality of product was approached as a dynamic-programming problem. This article presented a method for determining minimum cost allocations of screening inspection effort to satisfy both a quality requirement and a linear cost of outgoing defectives. A major merit of this model was that the computational procedure allowed rapid desk-calculator solutions to this problem. At this point it seems appropriate to mention some approaches that are based on ideas that have been expressed in the articles covered so far. White (28) proposed inspection plans for an ordered production process consisting of (N-1) manufacturing stages and a final inspection stage; Pruzan and Jackson (29) assumed that the number of defectives in a given inspection was not recorded, or alternatively, the number of nondefective items remaining after the inspection was known; Brown (23) proposed to first estimate parameters of serial N-stage production system which affected final quality, and then found the optimal inspection and disposition procedures which would maintain a given quality standard; Dietrich (24) proposed a systematic method of determining sampling policy throughout a multisystematic method of determining sampling policy throughout a multi-stage production process where the fraction defective at each stage of the process was considered constant during the processing of any lot, but varied from lot to lot with a stochastically independent beta random variable. Britney (22) used the idea of Lindsay and Bishop (27) proposing a model for obtaining a minimum cost defective screening program in an n-stage nonserial production process. Total cost included: a) cost of appraisal, b) cost of detection and correction of the defects and c) costs of external and internal failures. Defectives, screened out at some stage during the process, entered the process again after being repaired. The author used branch and bound methods which proved to be efficient procedures given simple and sharp bounds. The primary contribution of this and earlier models was in the form of recommendations of courses of action for quality control screening. For nonserial production processes operating under quasi-concave cost structures, optimal screening programs remained extreme point solutions. Serial production systems have been viewed as a special case of this model. For serial production systems under linear inspection cost structure, the extreme point solutions paralleled and supported the earlier findings of Lindsay and Bishop (27) and White (28). So the model was an extension to nonserial production systems and a special class of nonlinear cost structures. Ercan and Hassan (30) outlined the integrated system approach, taking into account the interrelations among incoming quality level, the process quality limits at each stage of production and an outgoing quality level. The model has been restricted to the development of single sampling plans where inspection was by attribute, assuming the number of pieces in the lot was fixed and the sample size was the same for incoming and outgoing inspection for each stage of production. The minimization problems have been converted to minimization problems without constraints using Lagrangian functions and then solved by an iterative procedure. Taulananda (26) considered two cases of interrelated sampling plans. The first one was the single-stage, single-component manufacturing system (SSSC), where a lot of one kind of material or component comes into a production department; an operation in one stage was performed and then the finished part was inspected before leaving the production department. The second case was the multistage, single-component manufacturing system (MSSC), in which sequence of SSSC subsystems were involved. An algorithm was proposed which included a termination rule followed by a numerical example. The approach to the optimal solution was by a search scheme. The algorithm has been set to allow a solution by computer. The importance of this model was that it showed that not only the sampling plan could be selected but also the appropriate incoming quality level and average process fraction defective. The advantage of a total system approach should be obvious. However one drawback was the dependency upon a computer since the models are complex, and cannot be solved by analytical techniques.

Ercan, Hassan, Taulananda. Based on their previous work (30) the authors (25) dealt with a single stage manufacturing connected-unit situation. This situation arose when one kind of material procured was inspected and then processed through a production stage; an operation or a series of operations performed, and then the finished part was inspected. To obtain minimum single sampling plans for the single stage manufacturing system, the authors specified several loss functions. A computer program was written for the system and single sampling plans were obtained for cases where the lot size is fixed and inspection is by attribute.

Models for Process Control

The important contributions in the area are by Carter (33), Duncan (34) and Montgomery and Klatt (35), among others. The problem of process control is one of maintaining a production process in such a state that the output from the process conforms to the design specifications. As the process operates over time, it will be subject to changes which will cause the output quality to deteriorate. At some point it becomes less costly to stop and overhaul the process.

Girshik and Rubin. The authors (36) assumed that a machine has four possible states: two performance levels and two states occuring during overhauls. The model was solved first by computing the equilibrium distribution for an arbitrary stopping rule and then minimizing the expected cost with regard to the variable parameters of the rule.

Bather. The Girshik and Rubin approach (36) was severely restricted, because of the introduction of equilibrium distributions. Bather (37) has tried to avoid it by using dynamic programming. It was assumed that the production control was carried out by means of a sequence of decisions taken at regular times, or alternatively, control was done by a continuous inspection. The analysis did not include an investigation of the optimal sampling plan and exact solutions were difficult to attain.

Carter. The author (33) extended the Bayesian approach in the design of the control procedure. Using dynamic programming, his analysis showed how the constant optimal sample size could be found and how the optimal decision could be made based on the outcome of the sample. His model was quite similar to that of Bather (37), the principal difference was that the model allowed for determination of the sample size while Bather's did not. Some extensions of the above models were suggested. Tiago de Oliviera and Littauer (38) have developed a procedure for economically determining the constant interval between samples for maintaining statistical stability. Duncan (19) and Knappenberger and Grandage (39) have considered the optimal determination of an $\overline{X}$ Chart.

Montgomery and Klatt (35) have determined the constant optimal interval between samples assuming that the time between occurrences is exponentially distributed.

Summary

All the aforementioned developments have laid a solid foundation for economically based quality control procedures. The methods that have been described above have been categorized according to the system each method tackled. In Table 1 the features of the approaches that have been described are summarized as to the costs, profile (deterioration character), method of solution and other characteristics. In Table 2, a summary of the ease of use of each model is presented.

(1) Department of Defense. Military Standard 105A: Sampling Procedures and Tables for Inspection by Attributes. Washington: U.S. Government Printing Office.

(2) Dodge, N.F. and Romig, H.G. "Single and Double Sampling Inspection Tables". Bell System Technical Journal, 20 (January 1941), 1-61.

(3) Enell, J.W. "Which Sampling Plan Should I Choose?" Industrial Quality Control, 10, 6 (May 1954), 96-100.

(4) Martin, C.A. "The Cost Breakeven Cost in Attribute Sampling". Industrial Quality Control, September 1964, 137-144.

(5) Shahnazarian, T.E. "Selection of Economical Quality Levels for Multi-Component Assemblies". Industrial Quality Control, October 1965, 178-183.

(6) Romig, H.G. Sampling Inspection Tables. John Wiley, 1959.

(7) Ailor, R.H. "Economic Acceptance Sampling Plans for a Mixture of Variables and Attributes". Ph.D. dissertation, Virginia Polytechnic Institute, 1973.

(8) Guthrie, D. Jr. and Johns, M.V., Jr. "Bayes Acceptance Sampling Procedures for Large Lots". Annals of Mathematical Statistics, 30, 4 (December 1959), 896-925.

(9) Hald, A. "The Determination of Single Sampling Inspection Plans Based on Prior Distributions and Costs". Technometrics, 2, 3 (August 1950), 275-340.

(10) Heermans, J.H. "Determination of Optimal In-Process". Industrial Quality Control, 18, 12 (June 1962), 22-37.

(11) McCaslin, J.A. "Development and Analysis of a Multi-Attribute Quality Control Cost Model". M.Sc. dissertation, Virginia Polytechnique Institute, 1973.

(12) Penkov, B. and Theodoresen, R. "On the Game Theoretical Formulation of Sampling Plans". Matrika, 11, 2 (1966), 122-126.

(13) Smith, B.E. "The Economics of Sampling Inspection". Industrial Quality Control, March 1965, 453-458.

(14) Wald, A. Statistical Decision Functions. New York, John Wiley and Sons, 1950.

(15) Satterthwaite, F.E. and Grad, B. The Choice of Lot Inspection Plans on the Basis of Cost. General Electric Company, Report No. 135, April 1949.

(16) Satterthwaite, F.E. A New Continuous Sampling Inspection Plan Based on an Analysis of Costs. General Electric Company, Report No. 130, February 1949.

(17) Breakwell, John V. "The Problem of Testing for the Fraction of Defectives". Journal of the Operations Research Society of America, 2, 1 (February 1954), 59-69.

(18) Breakwell, John V. "Economically Optimum Acceptance Tests". Journal of American Statistical Association, 51 (June 1956), 243-256.

(19) Duncan, Acheson J. "The Economic Design of X-Charts When There is a Multiplicity of Assignable Causes". Journal of the American Statistical Association, 66 (March 1971), 107-121

(20) Cowden, D.T. Statistical Methods in Quality Control. New Jersey: Prentice Hall, Inc., 1957.

(21) Guenther, W.C. "On the Determination of Single Sampling Attribute Plans Based Upon a Linear Cost Model and a Prior Distribution". Technometrics, 13, 3 (August 1971), 483-498.

(22) Britney, R.R. "Optimal Screening Plans for Non-Serial Production Systems". Management Science, 18, 9 (May 1972), 550-559.

(23) Brown, E.D. Some Mathematical Models of Inspection Along a Production Line. A Technical Report No. 35, Operational Research Center, M.I.T., May 1968.

(24) Dietrich, D.L. "A Bayesian Quality Assurance Model for a Multistage Production Process". Ph.D. thesis, University of Arizona, 1971.

(25) Ercan, S.S., Hassan, M.F. and Taulananda, A. "Cost Minimizing Single Sampling Plans with AIQL and AOQL Constraints". Management Science, 20, 7 (March 1974), 1112-1120.

(26) Taulananda, A. "Design of Optimal Manufacturing Quality Assurance Systems Using Dynamic Programming". Ph.D. dissertation, Illinois Institute of Technology, 1973.

(27) Lindsay, G.F. and Bishop, A.B. "Allocation of Screening Inspection Effort -- A Dynamic Programming Approach". Management Science, 10, 2 (January 1964), 342-352.

(28) White, L.S. "The Analysis of a Simple Class of Multistage Inspection Plans". Management Science, 12, 9 (May 1966), 685-693.

(29) Pruzan, P.M. and Jackson, J.T.R. "A Dynamic Programming Application in Production Line Inspection". Technometrics, 9, 1 (February 1967), 73-81.

(30) Ercan, S.S., Hassan, M.F. "Cost-Affected Single Sampling Plans for Single-Stage Multicomponent Unit Manufacturing Situation". Proceedings of the 11th American Meeting of the Institute of Management Sciences, Los Angeles, California, 1970.

(31) Duncan, A.J. Quality Control and Industrial Statistics, Third edition, Richard D. Irwin, Inc., Illinois, 1970.

(32) Everett, H. "Generalized Lagrange Multiplier Method for Solving Problems of Optimum Allocation of Resources". Operations Research, 11 (1963), 399-417.

(33) Carter, L.F. "A Bayesian Approach to Quality Control". Management Science, 18, 11 (July 1972), 647-657.

(34) Duncan, Acheson, J. "The Economic Design of $\bar{X}$-Charts Used to Maintain Current Control in Process". Journal of the American Statistical Association, 51, 274 (June 1956), 228-242.

(35) Montgemery, D.C. and Klatt, P.J. "Economic Design of T^2 control Charts to Maintain Current Control of a Process". Management Science, 19, 1 (September 1972), 76-98.

(36) Girshik, M.A. and Rubin, H. "A Bayes Approach to a Quality Control Model". Annals of Mathematical Statistics, 28 (1952), 114-125.

(37) Bather, J.A. "Control Charts and the Minimization of Costs". Journal of Royal Statistics (Ser. B), 25 (1963), 49-80.

(38) Tiago de Oliviera, J. and Littauer, S.B. "Double Limit and Run Control Charts". Revue de Statistique Appliquée, 13,2 (1965),20-31

(39) Knappenberger, II, Allan and Grandage, A.H.E. "Minimum Cost Quality Control Tests". AIIE Transactions, March 1959, 24-32.

Contributors	Costs*	Profile	Sampling Plan		Method of Solution **
			Att.	Var.	
Enell, 1954	1,8,9,11		X		A
Martin, 1964	1,8,9		X		A
Shanazarian, 1965	1,7,8,11		X		A
Breakwell, 1954	1,8,9		X	X	A
Cowdan, 1957	1,12,13		X	X	A
Hald, 1960	1,9		X		S
Heermans, 1962	1,6,11		X		S
Penkov-Theodoresen, 1966	1,5,8,9		X	X	G
Guenther, 1971	1,8,9,10,11		X		A
Beightler-Mitten, 1964	1,13		X		D
Lindsay-Bishop, 1964	1,11,13		X		D
Ercan-Hassan, 1970	1,7,11		X		L
Britney, 1972	1,8,11		X		S
Taulananda, 1973	1,11,13		X		S
Ercan-Hassan-Taulananda, 1974	1,7,11		X		L
Girshik-Rubin, 1952	1,3,5	Descrete 2 states			A
Bather, 1963	1,13				D
Carter, 1972	1,2,3	normal		X	D
Menipaz, 1975	1,2,3,4,5,6	any	X	X	F,D

* Cost Key

1. Cost of sampling
2. Cost per unit of lot time
3. Cost of adjusting a faulty process
4. Cost of adjusting a controlled process
5. Cost of continuous inspection
6. Cost of switching
7. Cost of shipping defectives
8. Cost of defective that enters production
9. Rejection cost of a good lot
10. Cost of defectives in in rejected lot
11. Cost of replacing defective
12. Cost of wrong decisions
13. Operating Costs

** Method of Solution Key

A - Analytical
D - Dynamic Programming
F - Differentiation
G - Game Theory
L - Lagrange Multipliers
S - Search Techniques

Table 1(a) Features of Comparison of Current Available Policies

	Compu-terized	Consider-ations of AQL or Acceptance Number	Consider Sample Size	Interval Between Samples		Net Present Value	Var-iable Costs (Over time)	Consider	
				Const.	Var.			Control Limits	Assemblies
Enell, 1954		X							
Martin, 1964		X							
Shanazarian, 1965		X							X
Breakwell, 1954			X						
Cowdan, 1957			X	X				X	
Hald, 1960			X						
Heermans, 1962	X		X						
Penkov-Theodoresen, 1966		X	X						
Guenther, 1971		X	X						
Beithtler-Mitten, 1964	X		X						
Lindsay-Bishop, 1964		X	X						
Ercan-Hassan, 1970		X	X					X	
Britney, 1972									
Taulananda, 1973	X		X					X	X
Ercan-Hassan-Taulananda, 1974	X		X						
Girshik,Rubin, 1952				X					
Bather, 1963		X		X					
Carter, 1972			X	X		X			X
Menipaz, 1975					X	X	X		X

Table 1(b) Features of Comparison of Current Available Policies

Contributors	Data Requirements	Manual Operation Possible	Computer Program Available	Difficulty in Updating	Operator Proficiency Level
Enell, 1954	L	X		L	P
Martin, 1964	M	X		M	Q
Shazanarian, 1965	M	X		M	Q
Breakwell, 1954	M	X		M	Q
Cowdan, 1957	M	X		C	D
Hald, 1960	C	X		C	D
Heermans, 1962	C			C	D
Penkov-Theodoresen, 1966	M	X		M	D
Guenther, 1971	M	X		M	D
Beightler-Mitten, 1964	C	X	X	C	D
Lindsay-Bishop, 1964	M			C	D
Ercan-Hassan, 1970	C			C	D
Britney, 1972	M	X		M	D
Taulananda, 1973	C		X	C	D
Ercan-Hassan-Taulananda, 1974	C		X	C	D
Girshik-Rubin, 1952	M	X		C	D
Bather, 1963	C			C	D
Carter, 1972	C	X		C	D
Menipaz, 1975	M	X	X	L	D

Table 2: Ease of Use Comparison Between Current Available Policies and the Suggested Approach.

Symbol Keys

Data Requirements:

L - Little
M - Moderate
C - Considerable

Difficulty of Updating

L - Low
M - Moderate
C - Considerable

Operator Proficiency:

P - Process operator
Q - Quality control technician
D - Degreed professional

Figure 1

Different Economic Models for Quality Control

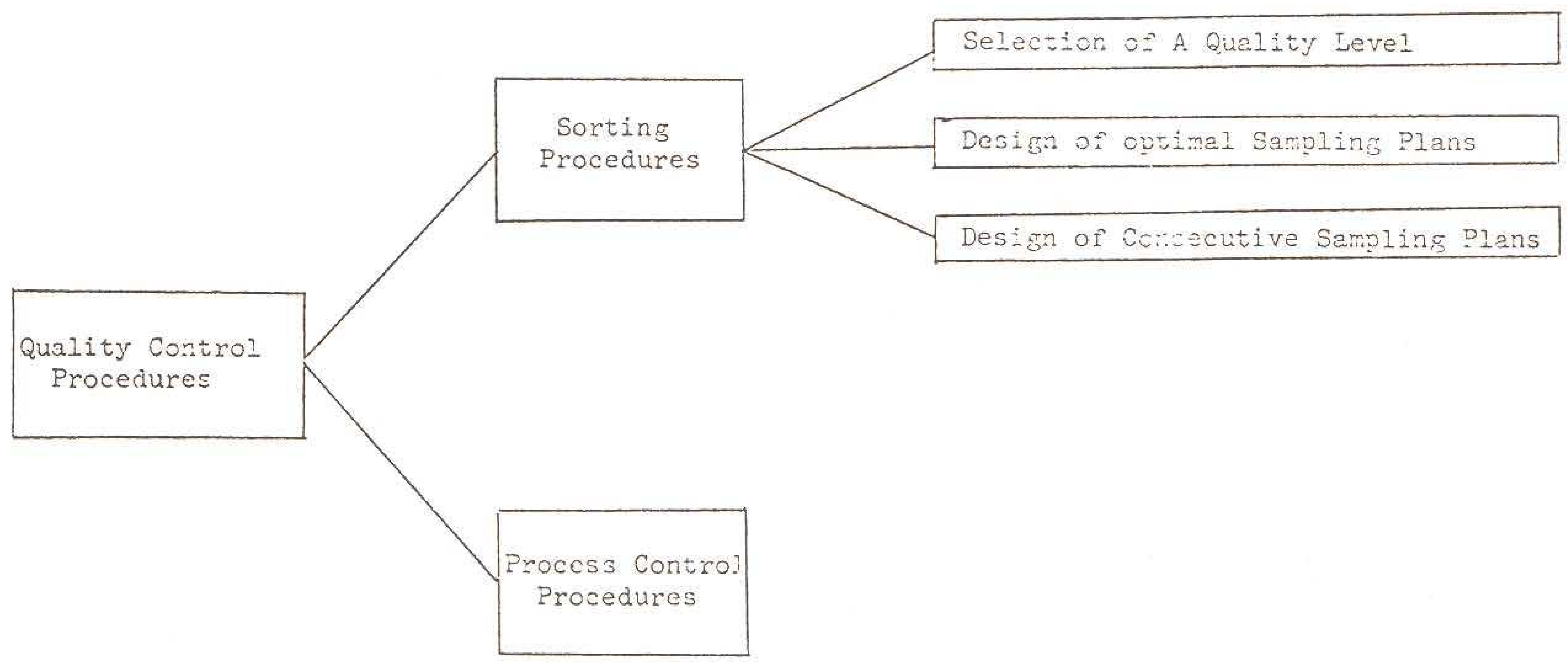
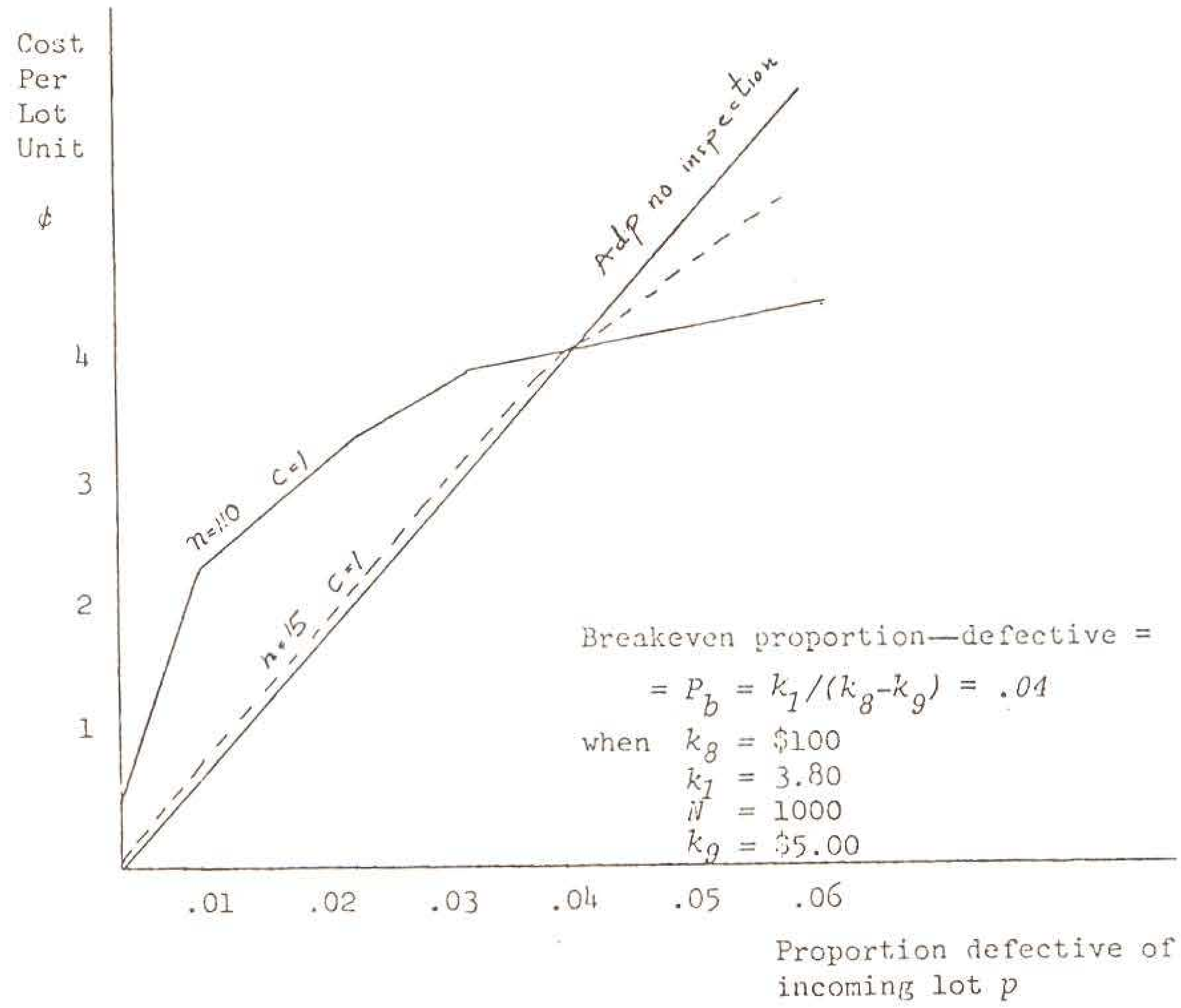


Figure 2

Inspection Cost Curves Comparing Sampling

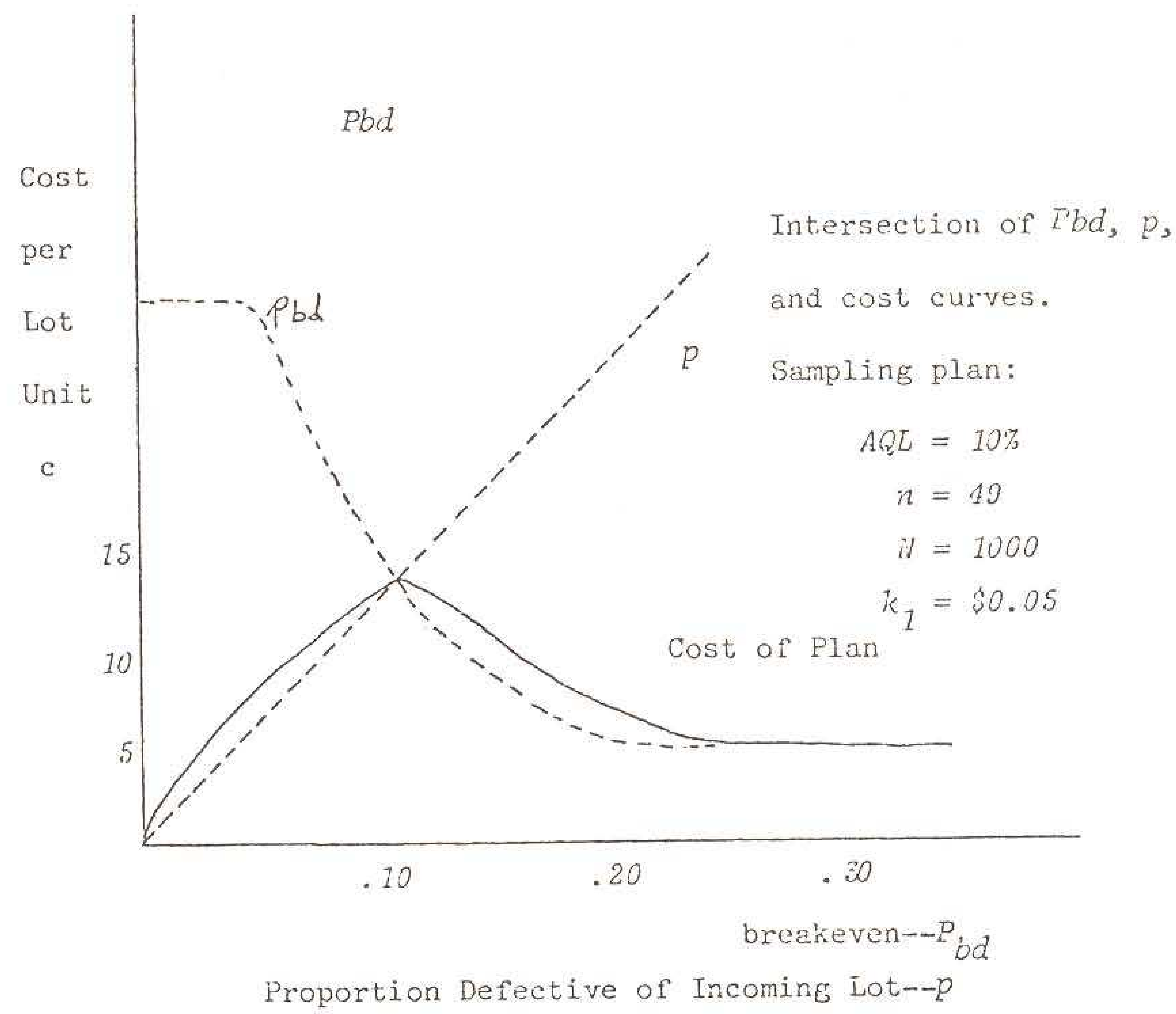
Plans and Curve of No Inspection



Source: J. W. Enell, "Which Sampling Plan Should I Choose?" Industrial Quality Control, Vol. 10, No. 5 (May 1954), p. 137.

Figure 3

Destructive Inspection Plan Cost Curve Showing Breakeven Point



Source: J. W. Enell, "Which Sampling Plan Should I Choose?" Industrial Quality Control, Vol. 10, No. 5 (May 1954), p. 141