Elementary shortest path problems for a multiple-trip vehicle routing problem

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We consider a rich vehicle routing problem (RVRP) encountered at a Belgian transportation company in charge of servicing supermarkets and hypermarkets belonging to a franchise. The problem at hand can be considered as a *one-to-many-to-one pick-up and delivery problem*, where there is a single depot from which all delivery customers are served and to which every pick-up demand of backhaul customers must be carried back [3]. On a given route, a vehicle can visit a backhaul customer only after all its delivery customers for the route are visited, where a customer is either a delivery or a backhaul customer, but not both. Every customer must be served within its associated time window, but it is not possible to serve every customer with every available vehicle, since the company's own fleet consists of different types of vehicles according to which the capacity varies. Therefore, our problem falls also into the category of *heterogeneous fleet vehicle routing problems with customer-vehicle incompatibilities* [1]. In this problem, multiple trips are allowed so that each vehicle can be assigned to several routes. In addition, a vehicle can begin servicing the customers only when the shift of the assigned driver starts which leads us to a multiple-trip RVRP with driver shifts. The service time of a vehicle can exceed the shift duration by bearing an overtime cost. In addition to the own fleet, the company has also the possibility to hire vehicles, which can be used for a fixed period by starting the service at any desired time. A fixed reservation cost and distance and time based variable costs are incurred in the case of a hired vehicle, while only distance based variable costs and overtime costs are incurred in the case of a vehicle of the own fleet.

We formulate a binary integer linear program to model our problem. In order to solve the problem, we first relax the binary restrictions on the assignment variables and develop a column generation procedure, where we obtain two pricing problems, one for vehicles of the company's own fleet and the other one for hired vehicles. The pricing problem for each vehicle of the company's own fleet can be formulated as an elementary shortest path problem with resource constraints (ESPPRC), which can be solved using a dynamic programming algorithm [2, 4]. However, since a hired vehicle can start the service at any point in time and is paid based on its total travel time, the second pricing algorithm has to take into account an infinite number of Pareto-optimal states. In order to solve this sec-

ond pricing problem which can be formulated as an ESPPRC as well, we develop piecewise linear time functions depending on a variable start time at the depot and develop appropriate dominance rules to discard feasible paths that cannot lead to the optimal solution. Finally, we present preliminary computational results.

References

[1] Ceselli, A., G. Righini, M. Salani, A column generation algorithm for a rich vehicle-routing problem, Transportation Science 43 (2009) 56–69.

[2] Feillet, D., P. Dejax, M. Gendreau, C. Gueguen, An exact algorithm for the elementary shortest path problem with resource constraints: application to some vehicle routing problems, Networks 44 (2004) 216–229.

[3] Gutiérrez-Jarpa, G., G. Desaulniers, G. Laporte, V. Marianov, A branch-and-price algorithm for the vehicle routing problem with deliveries, selective pickups and time windows, European Journal of Operational Research 206 (2010) 341–349.

[4] Righini, G., M. Salani, New dynamic programming algorithms for the resource constrained elementary shortest path problem, Networks 51 (2008) 155–170.