Approximation Algorithms for Multi-Dimensional Vector Assignment problems

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1 Introduction

We consider a multi-dimensional assignment problem motivated by an application arising in the semi-conductor industry. Formally, the input of the problem is defined by m disjoint sets $V_1, \ldots, V_m$, where each set V_k contains the same number n of p-dimensional vectors with nonnegative integral components, and by a *cost function* $c(u) : \mathbb{Z}_+^p \to \mathbb{R}_+$. A (feasible) m-tuple is an m-tuple of vectors $(u^1, u^2, \ldots, u^m) \in V_1 \times V_2 \times \ldots \times V_m$, and a *feasible assignment* for $V_1 \times \ldots \times V_m$ is a set A of n feasible m-tuples such that each element of $V_1 \cup \ldots \cup V_m$ appears in exactly one m-tuple of H_m. We define the component-wise maximum operator $\vee$ as follows : for every pair of vectors $u, v \in \mathbb{Z}_+^p$,

$$u \vee v = (\max(u_1, v_1), \max(u_2, v_2), \ldots, \max(u_p, v_p)).$$

Now, the cost of an m-tuple $(u^1, \ldots, u^m)$ is defined as $c(u^1 \vee \ldots \vee u^m)$ and the cost of a feasible assignment A is the sum of the costs of its m-tuples : $c(H_m) = \sum_{(u^1,\ldots,u^m)\in A} c(u^1 \vee \ldots \vee u^m)$.

With this terminology, the *multi-dimensional vector assignment problem* (MVA-m or MVA for short) is to find a feasible assignment with minimum total cost for $V_1 \times \ldots \times V_m$. A case of special interest arises when all vectors in $V_1 \cup \ldots \cup V_m$ are binary 0–1 vectors and when the cost function is *additive*, meaning that $c(u) = \sum_{i=1}^p u_i$. We call *wafer-to-wafer integration problem* (WWI-m or WWI for short) this special case of MVA-m. In this paper, we investigate how closely the optimal solution of MVA-m can be approximated by polynomial-time approximation algorithms.

1.1 Wafer-to-wafer integration and related work

The motivation for studying the WWI problem arises from the optimization of the wafer-to-wafer production process in the electronics industry. The wafer-to-wafer yield optimization problem has recently been the subject of much attention in the engineering literature, for example see papers by Reda, Smith and Smith [?], Taouil and Hamdioui [?], Taouil, Hamdioui, Verbree and Marinissen [?], Verbree, Marinissen, Roussel and Velenis [?], etc.

Our main objective in this paper is to study the approximability of the MVA problem and of the WWI problem (in the sense of [?]). Let us note at this point that the wafer-to-wafer integration problem is usually formulated in the literature as a maximization problem (since one wants to maximize the yield). However, we feel that from the approximation point of view, it is more appropriate to study its cost minimization version. Indeed, in industrial instances, the number of bad dies in each wafer is typically much less than the number of good dies. Therefore, it is more relevant to be able to approximate the (smaller) minimum cost than the (larger) maximum yield.

2 Our Results

Our results can be summarized as follows :

- We study two natural layer-by-layer heuristics called *sequential* and *heaviest-first* heuristics
- We show that both these heurisitics have finite worse-case ratio for every fixed m, when the cost function is montone and submodular. We are able to establish a better performance ratio when the cost function is additive.
- We show that even the case of WWI-3 with two nonzero entries per vector is APX-hard.