The Red-Blue Transportation Problem

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Consider the well-known Transportation Problem (TP): given is a set of supply nodes S, each with supply a_i $(i \in S)$, a set of demand nodes D, each with demand b_j $(j \in D)$, with $\sum_{i\in S} a_i = \sum_{j \in D} b_j$, and a bipartite graph $(S \cup D, E)$, with a given cost c_{ij} for each edge $(i, j) \in E$. The question is how to send the flow from supply nodes to the demand nodes such that total cost is minimal. In the present study, the problem is generalized by associating a color, either red or blue, to each supply node. Thus, the set of supply nodes is partitioned into two sets R (red) and B (blue) such that $S = R \cup B$, and $R \cap B = \emptyset$. The additional requirement is that the set of supply nodes that actually supply a demand node should all have the same color. In other words, a demand node is only allowed to receive flow from supply nodes that are either all red or all blue. The resulting problem, which is denoted from now on as Red-Blue TP (RBTP), clearly is a generalization of the transportation problem: if all supply nodes have the same color, the TP arises.

Particularly related to RBTP is the TP with Exclusionary Side Constraints (TPESC), another generalization of the TP. In TPESC, for each demand node $j \in D$, a set of pairs of supply nodes is given, denoted by $F_j = \{\{i_1, i_2\}|i_1, i_2 \in S\}$. The problem is to send the flow from supply to demand nodes at minimum cost, such that each demand node $j \in D$ only receives supply from at most one supply node for each pair of supply nodes present in F_j. Clearly RBTP is a special case of TPESC: any RBTP instance can be converted to TPESC by setting $F_j = \{\{i_1, i_2\}|i_1 \in R, i_2 \in B\}$.

Although the name TPESC was coined by Sun [4], the problem was in fact introduced by Cao [1]. Cao and Uebe [2] proposed a tabu search algorithm for solving transportation problems with non-linear sideconstraints, while Sun [4] discussed two branch-and-bound algorithms for TPESC. More recently, Goossens and Spieksma [3] have shown that TPESC is NP-hard, and becomes pseudo - polynomially solvable if the number of supply nodes is fixed. Furthermore, these authors studied TPESC with identical exclusionary sets: a pseudo-polynomial algorithm is provided for the case with two demand nodes, and NP-hardness is proven for the case with three demand nodes.

In the present contribution, the special case of RBTP is studied. We establish the problem's complexity, and we compare two integer programming formulations. Furthermore, a maximization variant of RBTP is presented, for which two $\frac{1}{2}$-approximation algorithms are given. This result naturally extends to a K-color Max RBTP variant, giving two $\frac{1}{K}$-approximation algorithms. Finally, a computational insight is given on the performance of the integer programming formulations and the approximation algorithms, by varying the problem size, the partitioning of the supply nodes, and the density of the problem.

References

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