The Budget-Constrained Min Cost Flow Problem

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In this paper we describe a problem that we define as the Budget-Constrained Minimum Cost Flow (BCMCF) problem. The BCMCF problem is a natural extension of the well-known Minimum Cost Flow (MCF) [2] problem, with a fixed cost related to the use of arcs and a budget constraint. Network flow problems often become hard when extra constraints are added. Ahuja and Orlin [1] discuss the constrained maximum flow problem with a budget constraint related to the cost of flow. Beasley and Christofides [3] study the resource constrained shortest path problem. Demgensky et al. [4] deal with a flow cost lowering problem with an upgrade budget to buy upgrade units on arcs. The BCMCF has, to the best of our knowledge, not been described in literature before. We show that the Accessibility Arc Upgrading Problem (AAUP) [5] is a special case of the BCMCF problem. We propose an exact solution method based on Lagrange relaxation and a heuristic approach based on variable neighborhood search.

Let us start by describing the BCMCF problem in detail. In this problem a flow has to be sent from a set of supply nodes or sources, through the arcs of a network, to a set of demand nodes or sinks. For each arc in the network, there is a cost per unit of flow over the arc, and a fixed cost associated to the use of the arc. Note that there might be several arcs connecting any specific pair of nodes. The problem is to find a minimal flow cost such that the sum of the fixed costs incurred by using the arcs to transport the flow does not exceed a given budget. We consider the uncapacitated as well as the capacitated case. Basically, this problem is a standard minimum cost flow problem in which an additional decision indicates which arcs are actually used to pass the flow, such that the cost for using these arcs does not exceed a fixed budget.

A relevant application of the BCMCF problem comes from the accessibility arc upgrading problem (AAUP) [5]. AAUP is a network upgrading problem in which resources have to be allocated in order to improve the accessibility to a set of vertices in a network. In the domain of rural road network planning, this problem arises when allocating resources to upgrade roads of a rural transport network, in order to improve the access that communities in small villages have to regional centres. More specifically, there is a directed connected graph $G = (V, E)$. The vertex set V represents the set of villages and regional centres. Each arc from E has a set of levels associated; for each level of the arc, there is an upgrading cost and a traversal cost associated. Thus, the flow cost through an arc depends on the level of the arc. There is a total budget B available to upgrade the level of some arcs. We can show that the AAUP is a special case of the BCMCF problem. We transform the network as described above such that we obtain a network representing an instance of the BCMCF problem. For each existing arc in the network, there is a set of possible upgrading levels. Therefore, for each existing arc, we define one new arc per possible upgrading level, connecting the same pair of vertices. Now, for each arc there is a traversal cost and a cost for using the arc (corresponding the upgrading cost). We can then solve the problem by finding a min cost flow on the transformed network, such that the sum of the fixed costs incurred by using some of the arcs that represent an upgraded level, does not exceed the given budget.

We have developed a variable neighborhood search (VNS) approach with strategic oscillation for solving the BCMCF problem, and applied it to a set of AAUP instances. The VNS algorithm exploits the underlying network flow structure of the AAUP when defining and evaluating neighborhoods. Experiments show good performance of the VNS algorithm. Further, we have constructed an exact solution method for BCMCF based on Lagrange Relaxation. A computational study is to be performed.

References

[1] Ahuja, R. K. and J. B. Orlin, A capacity scaling algorithm for the constrained maximum flow problem, *Networks*, Vol. 25, pp. 89-98, 1995.

[2] Ahuja, R. K., T. L. Magnanti, J. B. Orlin, *Network Flows: Theory, Algorithms, and Applications*, Prentice Hall, 1993.

[3] Beasley, J. E. and N. Christofides, An algorithm for the resource constrained shortest path problem, *Networks*, Vol. 19, pp. 379-394, 1989.

[4] Demgensky, I., H. Noltemeier, H.-C. Wirth, On the flow cost lowering problem, *European Journal of Operational Research*, Vol. 137, pp. 265-271, 2002.

[5] Maya, P., S. Coene, P. Goos, K. Sörensen, F. C. R. Spieksma, The accessibility arc upgrading problem, *European Journal of Operational Research*, Vol. 224, 3, pp. 458-465, 2013.