On a queue with customer deadlines

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In a typical queueing system, customers present themselves at some service facility to receive some kind of service, and – if they cannot be served immediately upon arrival – wait patiently in a queue until the server is available for them. In some cases, however, customers may leave the queue unserved if their time in the queue becomes too big. There may be various reasons for such behaviour. One of them is customer impatience (see, e.g., [?, ?, ?, ?]. In this case, it is the customer that takes the decision to abandon prematurely, e.g., because the customer does not like to wait any longer or because the customer has other tasks to attend. On the other hand, also the system itself may decide to remove customers from the queue if servicing those customers is deemed not to be useful any more after some time in the queue, e.g., because those customers would not arrive soon enough at their next destination if they had to wait any longer (see, e.g., [?]). In either case, the queueing system is special in the sense that customers may disappear from the system without ever reaching the service facility.

In our research, we make an attempt to analytically investigate this kind of situation by means of a simple queueing model. Specifically, we consider a discrete-time queueing system with one server and an infinite waiting room. The arrival process of new customers in the system is characterized by means of a sequence of i.i.d. non-negative discrete random variables; the service of each customer requires exactly one time slot. Furthermore, we assume that the deadlines of the customers may be different from one customer to another, but that they are statistically independent and geometrically distributed with parameter σ. Note that if $\sigma = 1$, the system reduces to a simple discrete-time queue without deadlines.

We analyze the queueing performance of the system, resulting in an exact yet complicated expression for the pgf of the number of customers in the system. From this result, we derive both (complicated) exact and (easier) approximate expressions for the mean system content, the mean customer delay and the deadline-expiration ratio. Another part of our research is devoted to an alternative approach in which we express all relevant performance measures of the system in the form of power series in the deadline parameter σ. In this way, we obtain much simpler, but also approximate, results. We believe that this power-series approximation is one of the main contributions of our research, and that this technique may be useful in the solution of other queueing models that lead to hard-to-solve functional equations. Finally, we compare the different approximations and illustrate our results by means of some numerical examples.

The main restriction of this work seems to be the assumption that the service times of the customers are deterministically equal to one slot each and that deadlines of the customers are i.i.d. and geometrically distributed. Future work will focus on generalizations of these assumptions.

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