MINES Saint-Étienne
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Vehicle routing problems with road-network information

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INSPIRING INNOVATION
SINCE 1816

Vehicle Routing Problems

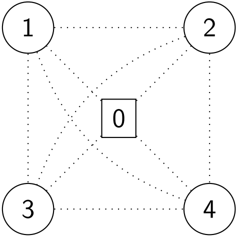
Given a complete graph $G = (V, A)$ with $V = \{0, \ldots, n\}$

- 0 is a depot where is available a fleet of vehicles of capacity Q
- nodes $\{1, \ldots, n\}$ are customers with a delivery demand q_i

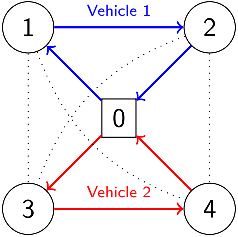
Given costs c_{ij} on arcs

Find a set of vehicle routes that serve all customers at a minimal total cost

Vehicle Routing Problems



(a) Graph G



(b) Solution

We call graph G customer-based graph

Arcs in G represent best paths in the original road-network

The first VRP

THE TRUCK DISPATCHING PROBLEM*

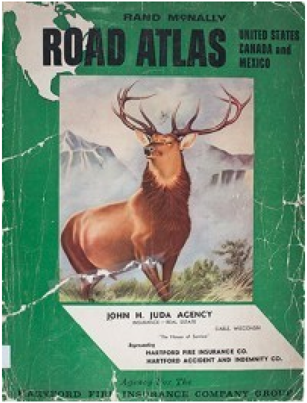
G. B. DANTZIG[1] AND J. H. RAMSER[2]

The paper is concerned with the optimum routing of a fleet of gasoline delivery trucks between a bulk terminal and a large number of service stations supplied by the terminal. The shortest routes between any two points in the system are given and a demand for one or several products is specified for a number of stations within the distribution system. It is desired to find a way to assign stations to trucks in such a manner that station demands are satisfied and total mileage covered by the fleet is a minimum. A procedure based on a linear programming formulation is given for obtaining a near optimal solution. The calculations may be readily performed by hand or by an automatic digital computing machine. No practical applications of the method have been made as yet. A number of trial problems have been calculated, however.

First paper published on the VRP

See : *G.B Dantzig, J.H. Ramser, The truck dispatching problem, Management Science, 1959*

The first VRP



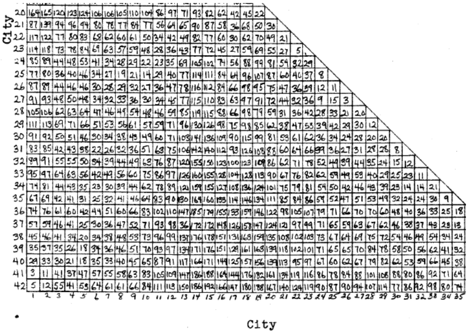
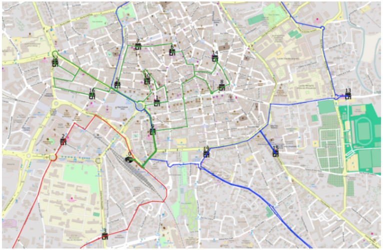


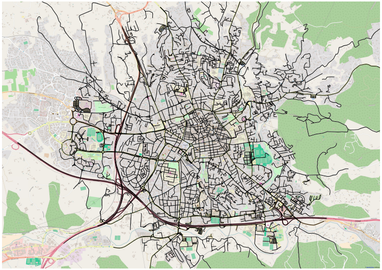
Figure: Geographical Information System (GIS)?
The Rand Mc Nally road atlas (1958)

VRPs nowadays



Trend 1: A lot of papers on urban distribution (city logistics)

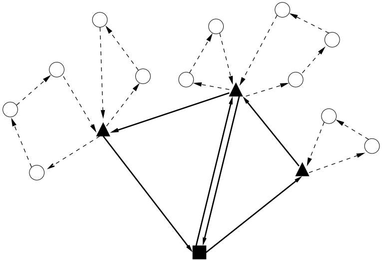
VRPs nowadays



Trend 2: Accurate data

- Geographic Information Systems (openStreetMap...)
- Traffic information (historical / real-time)
- Real-time monitoring

VRPs nowadays



Trend 3: Complex organizations / models

- Time constraints
- Multiple trips
- Multiple echelons (synchronization)
- Electric vehicles (range anxiety / recharging)
- Dynamic problem...

Outline of the presentation

PART I: New issues

1. Model granularity
2. Complex attributes
3. Multiple attributes

All these issues show the limits of the customer-based graph

PART II: Methodology

1. Multigraph
2. Road-network graph

1 Model granularity

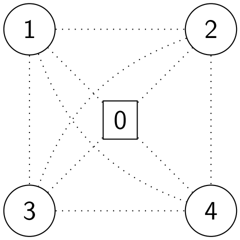
In the context of urban delivery, the distance between customers is often limited and the detail of operations (parking. . .) at customers becomes important.

Typical size of a parcel delivery tour

- 40 customers
- ≤ 5 minutes per customer, including service and traveling

See : *L. Bodin, V. Maniezzo, A. Mingozzi, Street routing and scheduling problems, in: Handbook of Transportation Science, 1999*

1 Model granularity



The classical model implicitly assumes:

- A unique and available parking location
- "Independence" of successive arcs in a tour

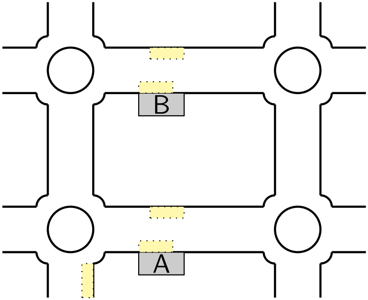
A unique and available parking location?

In practice :

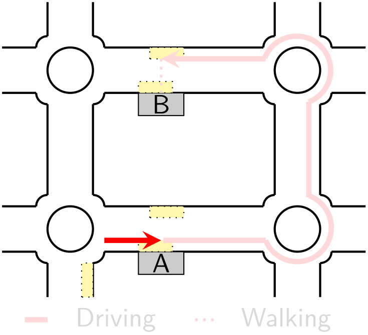
- parking in cities is complex
- several parking locations are possible
- some booking systems start being developed

See: *Z. Lang, E. Yao, W. Hu, Z. Pan, A vehicle routing problem solution considering alternative stop points, Procedia Social and Behavioral Sciences, 2014*

A unique and available parking location?



A unique and available parking location?



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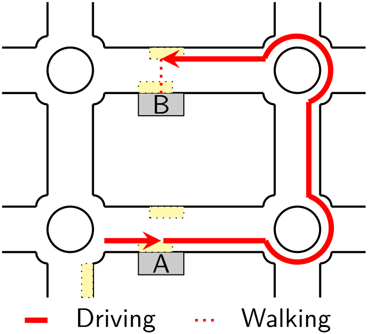
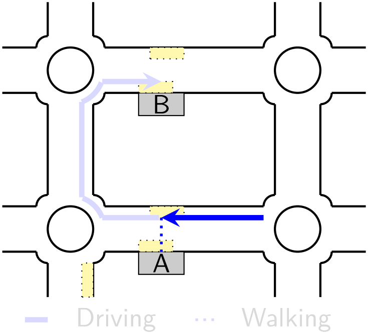
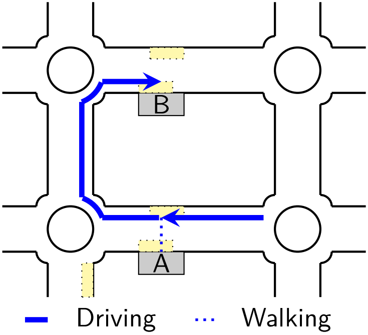


A unique and available parking location?



A unique and available parking location?



1 Model granularity

"Independence of successive arcs in a tour"?

Parking selection implies dependence between the ingoing and the outgoing arcs

This dependence also exists when some roads are subject to fees

See: *L. B. Reinhardt, M. K. Jepsen, D. Pisinger, The edge set cost of the vehicle routing problem with time windows, Transportation Science 2015.*

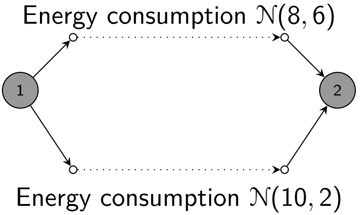
Some complications could arise with:

- Complex cost functions / constraints (fuel consumption minimization, congestion charges, etc.)
- Additional decisions
  - Breaks (driver working hour regulation)
  - Speed (speed optimization)

Not possible / not efficient (?) to precompute paths.

Complex cost functions / constraints

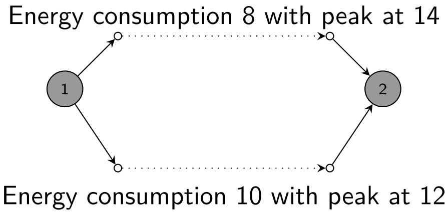
Illustration: Electric vehicle routing with stochastic energy consumption



Best path?

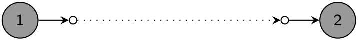
Complex cost functions / constraints

Illustration: Electric vehicle routing with deterministic energy consumption depending on street segment slopes



Best path?

Additional decisions: breaks (driver working hour regulation)



	Time intervals (min)		
	[0, 20[	[20, 40[	[40, 60]
Speed (km/h)	30	15	30

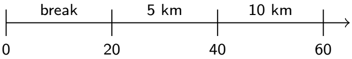
Break time: 20 minutes

Additional decisions: breaks (driver working hour regulation)

	Time intervals (min)		
	[0, 20[	[20, 40[	[40, 60]
Speed (km/h)	30	15	30

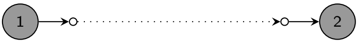
Break time: 20 minutes

Break at customer 1:



$\Rightarrow$ Customer 2 reached at time 60

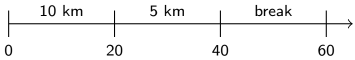
Additional decisions: breaks (driver working hour regulation)



	Time intervals (min)		
	[0, 20[	[20, 40[	[40, 60]
Speed (km/h)	30	15	30

Break time: 20 minutes

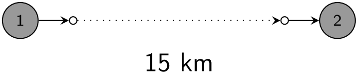
Break at customer 2:



⇒ Customer 2 reached at time 40, break finished at time 60

2 Complex attributes

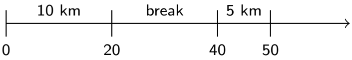
Additional decisions: breaks (driver working hour regulation)



	Time intervals (min)		
	[0, 20[	[20, 40[	[40, 60]
Speed (km/h)	30	15	30

Break time: 20 minutes

Break optimized on the road-network:



⇒ Customer 2 reached at time 50

See: *M. Chassaing, C. Duhamel, P. Lacomme. Time Dependent Capacitated Vehicle Routing Problem with Waiting Times at nodes. Odysseus, 2015.*

2 Complex attributes

Additional decisions: breaks (driver working hour regulation)

- In more complex networks, the path between customers 1 and 2 might even depends on the break time
- Also, the break time influences the previous / following parts of the route
- It is even possible that no solution exists with breaks at customers 1 or 2

Additional decisions: Speed

It is assumed that the decision-maker can control driver's speed to limit fuel consumption / pollution

Travel time / fuel consumption / pollution simple functions of the speed? No!

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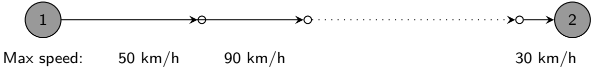


Additional decisions: Speed

In addition:

- Depending on this speed different paths will be followed
- The decision-maker might modify the speed at any node in the road-network

See : *J. Qian, R. Eglese. Finding least Fuel Emission paths in a network with time-varying speeds. Networks, 2014.*

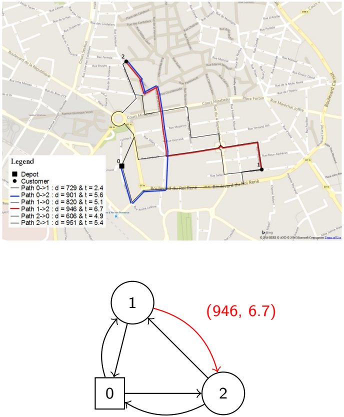
3 Multiple attributes

Examples of attributes:

- Distance.
- Travel time (not necessarily strongly correlated with distance).
- Energy consumption (electric vehicle), pollution, robustness, sightseeing, danger, tolls...

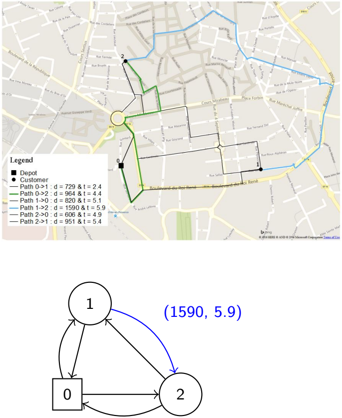
The best path is not necessarily the same for each attribute!

3 Multiple attributes



(a) Min-cost graph

(distance, time)



(b) Min-time graph

3 Multiple attributes

Some authors tried to evaluate numerically the consequences

T. Garaix, C. Artigues, D. Feillet and D. Josselin. Vehicle routing problems with alternative paths: an application to on-demand transportation. EJOR, 2010.

D. Lai, O.C. Demirag and J. Leung. A tabu search heuristic for the heterogeneous vehicle routing problem on a multigraph. Transportation Research Part E, 2016

H. Ben-ticha, N. Absi, D. Feillet, A. Quilliot, Empirical analysis for the VRPTW with a multigraph representation for the road network, Computers & Operations Research, 2017

Experiments show important increases of solution costs when using a customer-based graph, that can often exceed 10 %

Outline of the presentation

PART I: New issues

1. Model granularity
2. Complex attributes
3. Multiple attributes

All these issues show the limits of the customer-based graph

PART II: Methodology

1. **Multigraph**
2. **Road-network graph**

Outline of the presentation

Illustration with the VRP with Time Windows (VRPTW)

- Standard problem with 2 attributes: cost (distance) and time

Methodology

1. Model the road network with a multigraph.
   - One node is introduced for each customer, depot and other points of interest.
   - An arc is introduced for every efficient path between two nodes.
2. Apply directly solution methods on the road network.

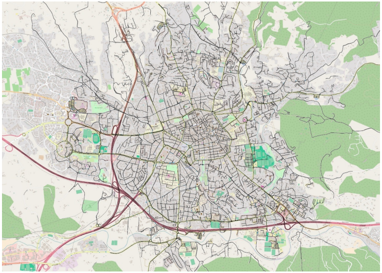
In both cases, contrary to a customer-based graph, no information is lost.

Main issues

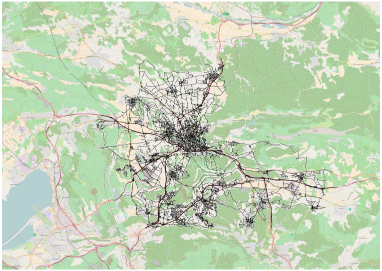
1. How to construct the multigraph? Size?
2. How to adapt exact solution schemes
   - in multigraphs?
   - in road-network graph?
3. Multigraph vs road-network graph?
4. How to adapt heuristic solution schemes
   - in multigraphs?
   - (in road-network graphs?)

Construction of the multigraph

Involve multi-objective shortest path problems: NP-hard



(a) 5437 nodes / 100 customers



(b) 19500 nodes / 100 customers

See: *H. Ben-ticha, N. Absi, D. Feillet, A. Quilliot, A solution method for the Multi-destination Bi-objectives Shortest Path Problem, submitted.*

Construction of the multigraph

Algorithm 1 `NaiveAlgorithm(s)`

$\mathcal{L} \leftarrow \{(s, 0, 0)\}$ //label definition: (last vertex,distance,time)
repeat
    Select $L = (i, d, t) \in \mathcal{L}$
    $\mathcal{L} \leftarrow \mathcal{L} \setminus \{L\}$
    for all j successor of i **do**
        $L' = (j, d + d_{ij}, t + t_{ij})$
        InsertWithDominance($L', \mathcal{L}$)
        // $L_1 \prec L_2 \Leftrightarrow i_1 = i_2$ and $t_1 \leq t_2$ and $d_1 \leq d_2$
    end for
until $\mathcal{L} = \emptyset$

Execute `NaiveAlgorithm` for each $s \in V$

Construction of the multigraph

Improvements:

- Implement a multi-objective multi-destination A* to guide the search:

  Select the label that minimizes *the detour* in distance **among all destinations**

- Stop the search once the key of the selected label is greater than the maximal detour **among all destinations**

Other improvements with Time Windows

- Consider only *reachable* customer nodes
- Only nodes that are apt to lead to a feasible path to a destination node should be considered

Construction of the multigraph

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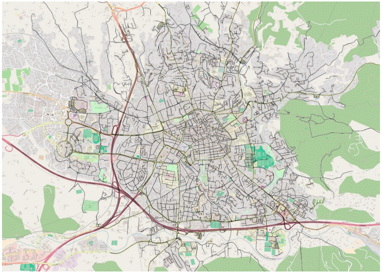
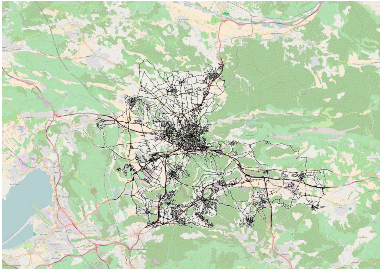
Construction of the multigraph



(a) 5437 nodes / 100 customers
`NaiveAlgorithm`: 5 seconds
`Multi-A*`: **2 seconds**



(b) 19500 nodes / 100 customers
`NaiveAlgorithm`: 400 seconds
`Multi-A*`: **30 seconds**

$\leq$ 4 arcs in parallel on average

Exact solution schemes
Multigraph

Branch-and-Price algorithms can easily be generalized

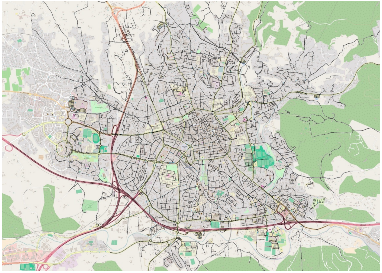
- **Master problem:** set partitioning problem
- **Pricing problem:** Elementary Shortest Path Problem with Resource Constraints on multigraph

  Solved using an adapted labelling algorithm: a label at some node is extended to all outgoing arcs

- **Branching rules:** standard branching rules

See: *H. Ben-ticha, N. Absi, D. Feillet, A. Quilliot, Empirical analysis for the VRPTW with a multigraph representation for the road network, Computers & Operations Research, 2017*

Computational results: Real instances



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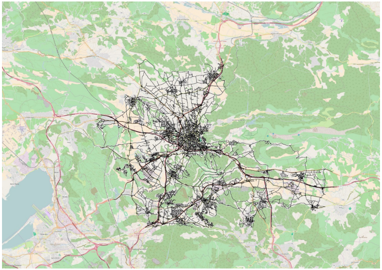

(b) 19500 nodes / 100 customers

Exact solution schemes
Multigraph

Computational results: Real instances

	$\|V\|$	$\|C\|$	#	min-cost graph CPU(s)	min-time graph CPU(s)	multigraph CPU(s)	gap cost (%) min-cost	gap cost (%) min-time
(a)	5437	25	1	0.2	0.3	1.7	**-3.4**	**-7.7**
			2	0.2	0.2	0.8	**-8.0**	**-6.3**
		50	1	1.9	4.3	13.4	**-2.3**	**-5.2**
			2	2.9	3.7	18.8	**-1.6**	**-4.4**
		75	1	23.4	21.2	131.4	**-0.5**	**-5.1**
			2	311.6	11.4	73.4	**-0.7**	**-5.9**
(b)	19500	25	1	1.5	0.2	1.2	**-6.6**	**-10.0**
			2	0.2	0.4	1.1	**-1.7**	**-8.7**
		50	1	4.0	12.6	22.7	**-0.1**	**-9.5**
			2	10.4	13.6	13.3	**-2.3**	**-8.3**
		75	1	599.1	1372.3	174.1	**-10.5**	**-4.1**
			2	55.4	23.6	102.6	**-0.9**	**-4.4**

Exact solution schemes
Road network

Column generation (fractional solution)

- The master problem is not modified
- The pricing algorithm is applied in the road-network graph
  - many nodes
  - a few arcs from each node
- The service is elementary but not the routes: crossroad nodes or arcs can be traversed many times...
- When extending a label to a customer, two labels are generated: one with service, one without service

See: *A. Letchford, S. Nasiri and A. Oukil. Pricing routines for vehicle routing with time windows on road networks. Computers & Operations Research, 2014.*

Exact solution schemes
Road network

Branch-and-price (integer solution)

- A fractional solution can be supported by an integer flow
- No simple way to deal with it
- Same difficulties in arc routing: *C. Bode and S. Irnich. Cut-First branch-and-price-second for the capacitated arc-routing problem, Operations research 2012.*

However it “never” happens (unlike to what is happening in arc routing problems)

Exact solution schemes
Road network

Branch-and-price (integer solution)

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Exact solution schemes
Road network

Branch-and-price:

- When the flow is fractional: branch on arc flow
- When the flow is integer:
  - **branch 1**: enumerate all the feasible routes in the subgraph supported by the flow and solve by IP
  - **branch 2**: impose to use an arc not in the subgraph supported by the flow

$$\sum_{(i,j)\in A\setminus\tilde{A}} \sum_{r\in\Omega} b_{ijr} x_r \geq 1$$

See: *H. Ben-ticha, N. Absi, D. Feillet, A. Quilliot, T. van Woensel, A branch-and-price Algorithm for the Vehicle Routing Problem with Time Windows on a road network, submitted*

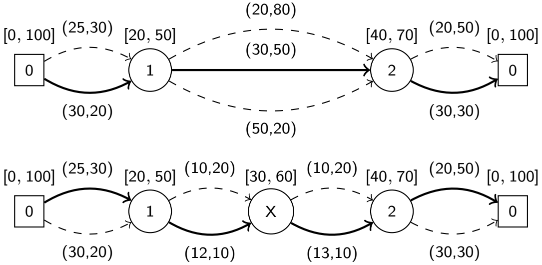
Multigraph versus road-network
Experiments

$\lvert V_{RN} \rvert$	$\lvert A_{RN} \rvert$	$\lvert C \rvert$		CPU_{MG}	CPU_{RN}	$\frac{CPU_{RN}}{CPU_{MG}}$
5437	10181	5	1	0.0	5.5	117
			2	0.1	4.2	64
			3	0.0	4.3	95
			4	0.1	8.8	149
			5	0.0	14.8	330
		10	1	0.1	11.9	128
			2	0.1	7.1	90
			3	0.1	6.0	87
			4	0.1	11.6	135
			5	0.1	25.6	337
		25	1	0.2	56.9	283
			2	0.2	51.3	252
			3	0.2	35.1	213
			4	0.4	111.8	285
			5	0.2	81.0	463
		50	1	1.0	113.5	114
			2	3.3	-	-
			3	2.1	147.6	70
			4	1.0	252.0	244
			5	17.4	-	-

—: instances not solved in 7200 seconds

Heuristic solution schemes
Multigraph

Local search operations (*e.g.*, an insertion, a removal) imply reoptimizing the selection of arcs.



Heuristic solution schemes
Multigraph

Arc selection is NP-hard

It can be managed by dynamic programming

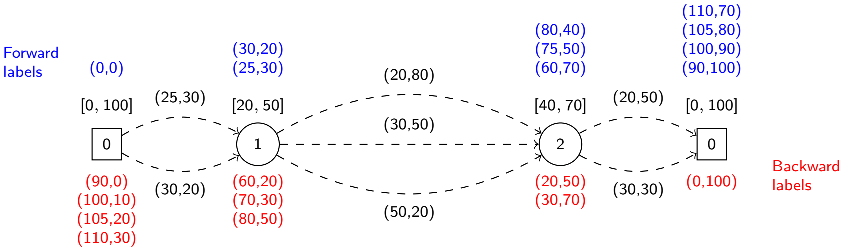
See: *T. Garaix, C. Artigues, D. Feillet and D. Josselin. Vehicle routing problems with alternative paths: an application to on-demand transportation. EJOR, 2010.*

Incremental techniques can be implemented to accelerate the method

See: *H. Ben-ticha, N. Absi, D. Feillet, A. Quilliot, T. van Woensel, Adaptive Large Neighborhood Search for VRPTW on multigraph, submitted.*

Heuristic solution schemes
Multigraph

1. Initially the best arcs are selected via dynamic programming (backward + forward)
2. Labels are stored
3. When a move is applied, these labels are used



Heuristic solution schemes

Multigraph

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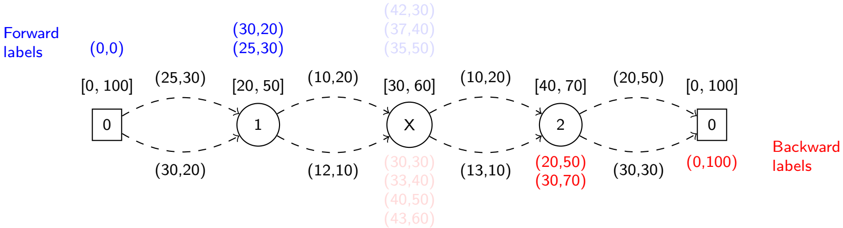

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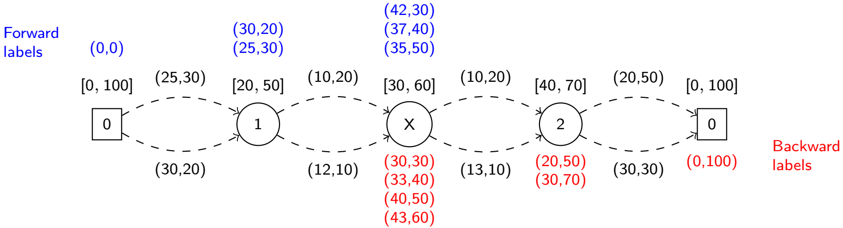

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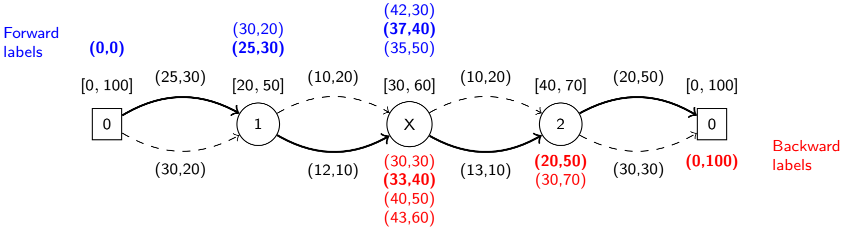

Conclusions

Recent VRPs often involve

- Urban distribution
- Accurate data
- Complex organization / models

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The number of papers investigating these issues is very limited...
...even if it has grown a lot recently!

Replacing the customer-based graph with a multigraph seems efficient, but is not always possible (or easy).

Replacing the customer-based graph with a road-network graph is not tractable yet.

Still a lot to do!

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Vehicle routing problems with road-network information

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