Designing a new Distribution Strategy for a Low-Value Goods Producer: A Practical Solution for the Inventory Routing Problem

El-Houssaine Aghezzaf⋆, Birger Raa⋆ and Wout Dullaert†

⋆Department of Industrial Management
Ghent University

†Institute of Transport and Maritime Management Antwerp
University of Antwerp

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Outline

Problem description

- A manufacturer producing around 250 different low-value products in Scandinavia.
- Over 100 customers from Belgium and The Netherlands delivered by a separate third party distributor for Belgium and for the Netherlands.
- Management was looking for a new distribution strategy to support their Benelux operations.
- Operating a single warehouse for the Benelux was an obvious part of that strategy.
- Aggregate demand rates per customer are stable, so cyclic planning is the appropriate approach.

Total logistics costs

The objective in the new distribution strategy is to minimize total system-wide costs.

Four different cost components are considered.

- Fixed cost per vehicle
- Variable transportation cost
- Fixed cost per reload and per delivery
- *Variable holding cost*

Because a cyclic approach is adopted, we will be talking about *cost rates*.

Assumptions

- The distributor has the responsibility of determining delivery frequencies to the customers.
- However, customers can restrict the delivery frequency by stating a limited storage capacity.
- Customers can also restrict the delivery frequency by specifying a maximum frequency.
- Frequencies are chosen such that for every customer, the time between deliveries is an integer number of weeks. A customer is thus always delivered on the same day of the week (but not necessarily every week).
- A driver/vehicle can drive no more than 8 hours per day.

Distribution patterns

A vehicle can be used to make different tours, and the different tours that are made by a vehicle can be repeated with different freqencies.

Thus, a *distribution pattern* is a multi-frequency multi-tour routing scheme for a single vehicle.

Cycle times

The cycle time of a distribution pattern is restricted by both vehicle and customer capacity restrictions.

$$T_{max} = \min\left(\min_{i=1..n} \frac{k_i \kappa}{\sum_{j \in S_i} d_j}, \min_{i=1..n} \min_{j \in S_i} \frac{k_i \kappa_j}{d_j}\right)$$

The cycle time is restricted by required driving, loading and unloading times, and by pre-specified maximum delivery frequencies.

$$T_{min} = \max\left(\sum_{i=1}^{n} k_i \left(T_{TSP(S_i+\Delta)} + t_\Delta + \sum_{j \in S_i} t_j\right), \max_{i=1..n} \max_{j \in S_i} \frac{k_i}{f_j}\right)$$

Cost rate of a distribution pattern

The cost rate of a distribution pattern (with the four components) depends on the cycle time T.

$$C = \psi + \frac{1}{T}\sum_{i=1}^{n} k_i \left(C_{TSP(S_i+\Delta)} + \varphi_\Delta + \sum_{j\in S_i} \varphi_j \right) + T\left(\sum_{i=1}^{n} \sum_{j\in S_i} \frac{\eta_j d_j}{2k_i} \right)$$

The trade-off between transportation costs and delivery costs on one hand and holding costs on the other hand, gives a theoretically optimal cycle time.

$$T_{EOQ} = \left(\frac{\sum_{i=1}^{n} k_i \left(C_{TSP(S_i+\Delta)} + \varphi_\Delta + \sum_{j\in S} \varphi_j \right)}{\sum_{i=1}^{n} \sum_{j\in S_i} \frac{\eta_j d_j}{2k_i}} \right)^{1/2}$$

Because low-value products are being considered, T_{EOQ} is infeasibly high, and the best feasible cycle time is T_{max}.

Solution approach I

- In the first phase of the project, the optimal location for the new Benelux-warehouse was determined.
- This was done with a center-of-gravity approach.
- The outcome was near the Belgian-Dutch border, along the E19 highway.

Solution approach II

In the second part of the project, the cyclic distribution planning from this warehouse needs to be organized. To solve this problem, four nested subproblems need to be solved.

- Partitioning customers over vehicles
- Partitioning customers over tours
- Determining tour frequencies
- Scheduling tours

Partitioning customers over vehicles

This is done by a savings heuristic.

1. The solution is initialized with a separate distribution pattern (DP) for each customer.
2. For each possible pair of DPs, a new DP is constructed that combines the two. If this results in a saving (i.e. the cost rate of the new DP is smaller than the sum of the cost rates of the two combined DPs), this new distribution pattern is kept.
3. The distribution pattern combination from Step 2 with the largest saving is selected. The two constituent DPs are removed from the solution and replaced by the new DP combining them.
4. Steps 2 and 3 are repeated as long as savings can be obtained by combining distribution patterns.

Partitioning customers over tours

Customers assigned to a vehicle have to be partitioned over tours. This sub-partitioning is done in the distribution pattern combination procedure, which is a savings heuristic in its turn.

- 2a A DP making all tours from both constituent DPs is constructed.
- 2b For every pair of tours in the DP, a new tour is constructed that combines the two, and a DP is constructed that makes this combined tour instead of the two original tours, plus all the other tours. If this results in a saving, this DP is kept.
- 2c The tour combination giving the largest saving is selected and the two combining tours are replaced by the larger combined tour.
- 2d Steps 2b and 2c are repeated as long as savings can be obtained by combining tours within the DP.

Determining tour frequencies

When customers are assigned to a vehicle and partitioned into different tours, tour frequencies need to be determined that minimize the cost rate of the distribution pattern for this vehicle. This is done by the following procedure.

- Initially, all tour frequencies are 1.
- There is one tour that restricts the maximal cycle time of the whole distribution pattern. The frequency of this tour is increased, after which a new schedule is made and the resulting cost rate is calculated. If the cost rate decreases, this step is re-iterated.

Scheduling the tours

To check the feasibility of a distribution pattern, a schedule needs to be made in which every tour i is made k_i times. If the length of this schedule is shorter than the maximal cycle time, the distribution pattern is feasible.

In the scheduling, tours have to be assigned to days, such that a vehicle does not have to drive more than 8 hours per day. This assignment is done by a greedy heuristic.

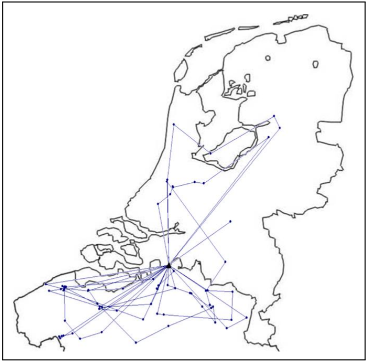
Local search and Multi-start

- At the end of the nested heuristic discussed above, local search is performed to further improve the solution.
- This four-level heuristic with local search is used in a multi-start framework, in which random rewards are assigned to customers at each iteration.
- Savings are calculated from ‘reduced’ cost rates, i.e. the regular cost rate minus the rewards of visited customers.

1. Problem description
2. Distribution patterns
3. Solution approach
4. **Computational results**
5. Conclusions

Two vehicles

To serve all Benelux-customers, two vehicles are needed.



13 different tours
12-week cycle

7 different tours
16-week cycle

Solution characteristics

Vehicle 1	Vehicle 2
56 tours in 48 days	69 tours in 53 days
12 days idle	27 days idle
24.2 traveling hours per week	22.1 traveling hours per week
90.6% loaded at dispatch	84.7% loaded at dispatch
1371 euro per week	1346 euro per week

Idle time

- There is a lot of idle time for both vehicles in the proposed solution.
- The fixed vehicle cost is always accounted for, regardless of whether the vehicle is traveling or idle.
- There are a number of opportunistic customers with irregular demands. These are not included in the cyclic framework.
- Idle time can be used to make deliveries to these opportunistic customers.
- Idle time = ‘spare capacity’

Conclusions

- A new, cyclic distribution strategy is designed for replenishing customers with stable demand rates.
- A generic solution framework is used that gives nicley balanced solutions in very reasonable computation times.
- Idle time in the schedule can be used for replenishing opportunistic customers.

- Future work may focus on extending the solution approach to richer problems (time windows, heterogenous vehicle fleet) and broadening the scope to include the replenishment of the warehouse in the cyclic approach.